Definition of Curvature

Let C be a smooth curve given by **r**(s), where s is the arc length parameter. The curvature at s is given by

$$K = \left\| \frac{d\mathbf{T}(s)}{ds} \right\| = \left\| \mathbf{T}'(s) \right\|.$$

Theorem

If C is a smooth curve given by **r**(t), then the curvature of C at t is given by

$$K = \frac{\left\| \mathbf{T}'(t) \right\|}{\left\| \mathbf{r}'(t) \right\|} = \frac{\left\| \mathbf{r}'(t) \times \mathbf{r}''(t) \right\|}{\left\| \mathbf{r}'(t) \right\|^3}$$

Theorem: Curvature in Rectangular Coordinates

If C is the graph of a twice-differentiable function given by $y = f(x)$, then the curvature at the point (x,y) is given by

$$K = \frac{|y''|}{[1 + (y')^2]^{(3/2)}}.$$

Theorem: Curvature in Polar Coordinates

If C is the graph of a twice-differentiable function given by $r = f(\theta)$, then the curvature at the point (r, θ) is given by

$$K = \frac{|2(r')^2 - rr'' + r^2|}{[(r')^2 + r^2]^{(3/2)}}$$