Consider a solid region Q whose density at (x, y, z) is given by the density function ρ. The center of mass of a solid region Q of mass **m** is given by $(\bar{x}, \bar{y}, \bar{z})$, where

$$m = \iiint_Q \rho(x, y, z)dV$$

$$M_{yz} = \iiint_Q x\rho(x, y, z)dV$$

$$M_{xz} = \iiint_Q y\rho(x, y, z)dV$$

$$M_{xy} = \iiint_Q z\rho(x, y, z)dV$$

and

$$\bar{x} = \frac{M_{yz}}{m}, \ \bar{y} = \frac{M_{xz}}{m}, \ \bar{z} = \frac{M_{xy}}{m}$$

The quantities M_{yz}, M_{xz}, M_{xy} are called the first moments of the region Q about the yz-, xz-, and xy - planes.

The first moments for solids regions are taken about a plane, whereas the second moments for solids are taken about a line. The second moments (or moments of inertia) about x-, y-, z - axes are as follows.

$$I_x = \iiint_Q (y^2 + z^2)\rho(x, y, z)dV$$

$$I_y = \iiint_Q (x^2 + z^2)\rho(x, y, z)dV$$

$$I_z = \iiint_Q (x^2 + y^2)\rho(x, y, z)dV$$

For problems requiring the calculation of all three moments, use the additive property of triple integrals

$$I_x = I_{xz} + I_{xy}; \qquad I_y = I_{yz} + I_{xy} \ ; \quad I_z = I_{yz} + I_{xz}$$

where I_{xy}, I_{xz}, I_{yz} are as follows.

$$I_{xy} = \iiint_Q z^2\rho(x, y, z)dV$$

$$I_{xz} = \iiint_Q y^2\rho(x, y, z)dV$$

$$I_{yz} = \iiint_Q x^2\rho(x, y, z)dV$$