Let f be a function whose domain is a subset A of $\Re^n$ and with range contained in $\Re^m$. By this we mean that to each $\mathbf{x} = (x_1, x_2, ..., x_n) \in A$, f assigns a value $f(\mathbf{x})$, an m-tuple in $\Re^m$. Such functions f are called vector-valued-functions if m > 1, and scalar-valued functions if m=1. In general, the notation $\mathbf{x} \mapsto f(\mathbf{x})$ is useful for indicating the value to which a point $\mathbf{x} \in \Re^n$ is sent. We write $f : A \subset \Re^n \to \Re^m$ to signify that A is the domain of f (a subset of $\Re^n$) and the range is contained in $\Re^m$. We also use the expression f maps A into $\Re^m$. Such functions f are called functions of several variables if $A \subset \Re^n$, n>1.

When $f : U \subset \Re^n \to \Re$, we say that f is a real-valued function of n variables with domain U.

Definition: Graph of a Function

Let $f : U \subset \Re^n \to \Re$. Define the graph of f to be the subset of $\Re^{n+1}$ consisting of all the points $(x_1, x_2, ..., x_n, f(x_1, x_2, ..., x_n))$ in $\Re^{n+1}$ for $(x_1, x_2, ..., x_n)$ in U. Symbolically,

$$\text{graph } f = \{(x_1, ..., x_n, f(x_1, ..., x_n)) \in \Re^{n+1} \mid (x_1, ..., x_n) \in U\}$$

Definition: Level Curves and Surfaces

Let $f : U \subset \Re^n \to \Re$ and let $c \in \Re$. Then the level set of value c is defined to be the set those points $\mathbf{x} \in$ U at which $f(\mathbf{x}) = c$. If n = 2, we speak of a level curve and if n = 3, we speak of a level surface. Symbolically,

$$\{\mathbf{x} \in U \mid f(\mathbf{x}) = c\} \subset \Re^n.$$

Note that the level set is always in the domain space.

The open disk of radius r and center $\mathbf{x}_0$ is defined to be the set of all points $\mathbf{x}$ such that $\|\mathbf{x} - \mathbf{x}_0\| < r$. This set is denoted $D_r(\mathbf{x}_0)$, and is the set of points $\mathbf{x}$ in $\Re^n$ whose distance from $\mathbf{x}_0$ is less than r.

Definition: Open Sets

Let $U \subset \Re^n$. We call U an open set when for ever point $\mathbf{x}_0$ in U ther exists some $r > 0$ such that $D_r(\mathbf{x}_0)$ is contained within U; symbolically, we write $D_r(\mathbf{x}_0) \subset U$.