

**Definition:** Work Done by a Variable Force

If an object is moved along a straight line by a continuously varying force  $F(x)$ , then the work  $W$  done by the force as the object is moved from  $x=a$  to  $x=b$  is

$$W = \lim_{\|\Delta\| \rightarrow 0} \sum_{i=1}^n \Delta W_i = \int_a^b F(x) dx.$$

**Definition:**

The work  $W$  done by a constant force  $\mathbf{F}$  as its point of application moves along the vector  $\mathbf{PQ}$  is given by either of the following.

$$W = \|\text{proj}_{\mathbf{PQ}} \mathbf{F}\| \|\mathbf{PQ}\| \quad \text{or} \quad W = \mathbf{F} \cdot \mathbf{PQ}$$

**Definition:** Line Integral of a Vector Field

Let  $\mathbf{F}$  be a continuous vector field defined on a smooth curve  $C$  given by  $\mathbf{r}(t)$ ,  $a \leq t \leq b$ . The line integral of  $\mathbf{F}$  on  $C$  is given by

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T} ds = \int_a^b \mathbf{F}(x(t), y(t), z(t)) \cdot \mathbf{r}'(t) dt.$$

Note: The work done on an object in a force field  $\mathbf{F}$  is given by :  $W = \int_C \mathbf{F} \cdot \mathbf{T} ds$ .