

$$(13) \quad \int_0^4 \int_0^{(1/2)(4-x)} \int_0^{(1/4)(12-3x-6y)} dz dy dx \Rightarrow 0 \leq x \leq 4; 0 \leq y \leq \frac{1}{2}(4-x); 0 \leq z \leq \frac{1}{4}(12-3x-6y)$$

when $x = y = 0$, $z \leq (1/4)(12-3x-6y) = 3$, $\therefore 0 \leq z \leq 3$

since $4z = 12-3x-6y \Leftrightarrow y = (1/6)(12+3x-4z)$

when $y = 0$, $12+3x-4z = 0 \Leftrightarrow x = (-1/3)(4z-12)$

$$\therefore \iiint_Q dQ = \int_0^3 \int_0^{(-1/3)(4z-12)} \int_0^{(1/6)(12+3x-4z)} dy dx dz$$

$$(15) \quad \int_0^1 \int_y^1 \int_0^{\sqrt{1-y^2}} dz dx dy \Rightarrow 0 \leq y \leq 1; y \leq x \leq 1$$

by rearranging $0 \leq x \leq 1; 0 \leq y \leq x$

$$\therefore \iiint_Q dQ = \int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz dx dy$$

$$(19) \quad x = 4-y^2, z = 0, z = x \Rightarrow 0 \leq z \leq x; 0 \leq x \leq 4-y^2; -2 \leq y \leq 2$$

$$V = \int_{-2}^2 \int_0^{4-y^2} \int_0^x dz dx dy = \frac{256}{15}$$

$$(21) \quad x^2 + y^2 + z^2 = a^2 \Rightarrow z = \pm \sqrt{a^2 - (x^2 + y^2)}$$

$$\text{if } z = 0 \Rightarrow x^2 + y^2 = a^2 \Rightarrow y = \pm \sqrt{a^2 - x^2} \Rightarrow -a \leq x \leq a$$

Since $f(x,y,z)$ is symmetric we only need to find the volume in 1 quadrant and multiply by 8.

$$\therefore V = 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-(x^2+y^2)}} dz dy dx = 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-(x^2+y^2)} dy dx$$