(11 d)

$$V=\pi\int_0^2\left[(y^2-6)^2-(6-4)^2\right]dy=\pi\int_0^2\left[(y^4-12y^2+36)-4\right]dy=\pi\int_0^2\left[y^4-12y^2+32\right]dy$$

$$=\pi\left[\frac{1}{5}y^5-4y^3+32y\right]_0^2=\frac{192\pi}{5}$$

(13) $y_1=x^2,\qquad y_2=4x-x^2$

$$y_1=y_2\Leftrightarrow x^2=4x-x^2\Leftrightarrow 2x^2-4x=0\Leftrightarrow x^2-2x=0\Leftrightarrow x^2-4x+1=1\Leftrightarrow (x-1)^2=1$$

$$\Leftrightarrow (x-1)=\pm 1\Leftrightarrow x=1\pm 1$$

$\therefore\ x=0,\ 2$ (points of intersection)

(a)

$$V=\pi\int_0^2\left[(4x-x^2)^2-(x^2)^2\right]dx=\pi\int_0^2\left[(16x^2-8x^3+x^4)-x^4\right]dx=\pi\int_0^2\left[16x^2-8x^3\right]dx$$

$$=\pi\left[\frac{16}{3}x^3-2x^4\right]_0^2=\frac{32\pi}{3}$$

(b) $R=6-x^2;\qquad r=6-(4x-x^2)$

$$R^2=(6-x^2)^2=36-12x^2+x^4$$

$$r^2=(x^2-4x+6)^2=x^4-8x^3+28x^2-48x+36$$

$$R^2-r^2=36-12x^2+x^4-(x^4-8x^3+28x^2-48x+36)=8x^3-40x^2+48x$$

$$V=\pi\int_0^2\left[8x^3-40x^2+48x\right]dx=\pi\left[2x^4-\frac{40}{3}x^3+24x^2\right]_0^2=\frac{64\pi}{3}$$

(15) $y_1=x,\qquad y=3,\ \ x=0$ (revolve around the line $y=4$)

$0\le x\le 3$

$R=4-x,\qquad r=4-3=1$

$$V=\pi\int_0^3\left[(4-x)^2-1\right]dx=\pi\int_0^3\left[(16-8x+x^2)-1\right]dx=\pi\int_0^3\left[15-8x+x^2\right]dx$$

$$=\pi\left[15x-4x^2+\frac{1}{3}x^3\right]_0^3=18\pi$$

(17) $y_1=\dfrac{1}{x},\ \ y_2=0,\ \ x=1,\ x=4$

$R=4,\ \ r=4-\dfrac{1}{x}$

$$V=\pi\int_1^4\left[16-\left(4-\frac{1}{x}\right)^2\right]dx=\pi\int_1^4\left[\frac{8}{x}-\frac{1}{x^2}\right]dx$$

$$=\pi\left[8\ln x+\frac{1}{x}\right]_1^4=\pi\left[8\ln 4-\frac{3}{4}\right]$$