(35) $$\int \frac{1+\sin x}{\cos x}dx = \int \frac{1}{\cos x}dx + \int \frac{\sin x}{\cos x}dx = \int \sec x dx + \int \tan x dx$$

$$= \ln|\sec x + \tan x| + \left(-\ln|\cos x|\right) + C$$

$$= \ln\left|\frac{\sec x + \tan x}{\cos x}\right| + C = \ln|\sec x(\sec x + \tan x)| + C$$

(37) $$\int \frac{2t-1}{t^2+4}dt = \int \frac{2t}{t^2+4}dt - \int \frac{1}{t^2+4}dt$$

For the first integral, let $u = t^2 + 4 \Rightarrow du = 2tdt$

thus, $\int \frac{2t}{t^2+4}dt = \int \frac{du}{u} = \ln|u| + C = \ln|t^2+4| + C = \ln(t^2+4) + C_1$

For the second integral, observe that it is in the form of an arctan

(note that you will learn how to compute this integral when you learn trig - subsitution)

Thus, $-\int \frac{1}{t^2+4}dt = -\frac{1}{2}\arctan\frac{t}{2} + C_2$

$$\therefore \int \frac{2t}{t^2+4}dt - \int \frac{1}{t^2+4}dt = \ln(t^2+4) - \frac{1}{2}\arctan\frac{t}{2} + C$$

(39) $$\int \frac{-1}{\sqrt{1-(2t-1)^2}}dt = -\int \frac{dt}{\sqrt{1^2-(2t-1)^2}}dt$$

Note that this integral is the form $\int \frac{du}{\sqrt{a^2-u^2}} = u'\arcsin\frac{u}{a} + C$

$u = 2t-1 \Rightarrow \frac{du}{dt} = 2$

$$\therefore -\int \frac{dt}{\sqrt{1^2-(2t-1)^2}}dt = -\frac{1}{2}\arcsin(2t-1) + C$$

(41) $$\int \frac{\tan\frac{2}{t}}{t^2}dt = \int \left(\frac{1}{t^2}\right)\tan\frac{2}{t}dt$$

Let $u = \frac{2}{t} \Rightarrow du = -\frac{2}{t^2}$

$$\therefore \int \left(\frac{1}{t^2}\right)\tan\frac{2}{t}dt = -\frac{1}{2}\int \tan u du$$

$$-\int \tan u du = -\frac{1}{2}\left(-\ln|\cos u|\right) + C = \frac{1}{2}\ln\left|\cos\frac{2}{t}\right| + C$$