(43) $\int \frac{3}{\sqrt{6x-x^2}}dx$

Complete the square, thus, $6x-x^2=-(x^2-6x)=-(x^2-6x+9)+9=9-(x-3)^2$

$$\therefore \int \frac{3}{\sqrt{6x-x^2}}dx=3\int \frac{dx}{\sqrt{3^2-(x-3)^2}}=3\arcsin\frac{x-3}{3}+C$$

(45) $\int \frac{4}{4x^2+4x+65}dx=4\int \frac{dx}{4\left(x^2+x+\frac{65}{4}\right)}=\int \frac{dx}{\left(x^2+x+\frac{65}{4}\right)}$

By completing the square,

$$x^2+x+\frac{65}{4}=x^2+x+\frac{1}{4}-\frac{1}{4}+\frac{65}{4}=\left(x+\frac{1}{2}\right)^2+\frac{64}{4}=\left(x+\frac{1}{2}\right)^2+4^2$$

thus, $\int \frac{dx}{\left(x^2+x+\frac{65}{4}\right)}=\int \frac{dx}{\left(\left(x+\frac{1}{2}\right)^2+4^2\right)}=\frac{1}{4}\arctan\frac{\left(x+\frac{1}{2}\right)}{4}+C=\frac{1}{4}\arctan\frac{2x+1}{8}+C$

(49) $\frac{dx}{dx}=(1+e^x)^2 \Leftrightarrow dy=(1+e^x)^2dx \Leftrightarrow \int dy=\int (1+e^x)^2dx \Leftrightarrow y=\int (1+e^x)^2dx$

$$\Rightarrow y=\int (1+2e^x+e^{2x})dx=\int dx+2\int e^xdx+\int e^{2x}dx$$

$$=x+2e^x+\frac{1}{2}e^{2x}+C$$

(51) $(4+\tan^2 x)y'=\sec^2 x \Leftrightarrow (4+\tan^2 x)\frac{dy}{dx}=\sec^2 x \Leftrightarrow dy=\int \frac{\sec^2 x}{4+\tan^2 x}dx$

$$\Leftrightarrow \int dy=\int \frac{\sec^2 x}{4+\tan^2 x}dx=\int \frac{\sec^2 x}{2^2+(\tan x)^2}dx$$

Let $u=\tan x \Rightarrow du=\sec^2 x$

$$\therefore \int \frac{\sec^2 x}{2^2+(\tan x)^2}dx=\int \frac{du}{2^2+(u)^2}=\frac{1}{2}\arctan\frac{u}{2}+C=\frac{1}{2}\arctan\frac{\tan x}{2}+C$$

$$\therefore y=\frac{1}{2}\arctan\frac{\tan x}{2}+C$$