

$$(53) \quad \int_0^{\pi/4} \cos 2x dx = \left[\frac{1}{2} \sin 2x \right]_0^{\pi/4} = \frac{1}{2}(1-0) = \frac{1}{2}$$

$$(55) \quad \int_0^1 x e^{-x^2} dx$$

$$\text{let } u = -x^2 \Rightarrow du = -2x dx$$

Converting the limits of integration

$$u(0) = 0, \quad u(1) = -1$$

$$\therefore \int_0^1 x e^{-x^2} dx = -\frac{1}{2} \int_{u=0}^{u=-1} e^u du = -\frac{1}{2} [e^u]_0^{-1} = -\frac{1}{2} \left[\frac{1}{e} - 1 \right]$$

$$(57) \quad \int_0^4 \frac{2x}{\sqrt{x^2 + 3^2}} dx$$

$$\text{Let } u = x^2 + 9 \Rightarrow du = 2x dx$$

Converting the limits of integration

$$u(0) = 9, \quad u(4) = 25$$

$$\therefore \int_0^4 \frac{2x}{\sqrt{x^2 + 3^2}} dx = \int_{u=9}^{u=25} \frac{du}{\sqrt{u}} = 2 \left[u^{(1/2)} \right]_9^{25} = 2(5-3) = 4$$