

$$(29) \quad \int e^{2x} \sin x dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{2} \int e^{2x} \cos x dx$$

$$u = \sin x \quad v = \frac{1}{2} e^{2x}$$

$$du = \cos x dx \quad dv = e^{2x} dx$$

$$-\frac{1}{2} \int e^{2x} \cos x dx = -\frac{1}{2} \left\{ \frac{1}{2} e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x dx \right\} = -\frac{1}{4} e^{2x} \cos x - \frac{1}{4} \int e^{2x} \sin x dx$$

$$u = \cos x \quad v = \frac{1}{2} e^{2x}$$

$$du = -\sin x dx \quad dv = e^{2x} dx$$

$$\therefore \int e^{2x} \sin x dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x - \frac{1}{4} \int e^{2x} \sin x dx$$

$$\Leftrightarrow \int e^{2x} \sin x dx + \frac{1}{4} \int e^{2x} \sin x dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x$$

$$\Leftrightarrow \left(1 + \frac{1}{4}\right) \int e^{2x} \sin x dx = \frac{5}{4} \int e^{2x} \sin x dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x$$

$$\Leftrightarrow \int e^{2x} \sin x dx = \frac{4}{5} \left(\frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x \right) = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + C$$

$$(31) \quad y' = x e^{x^2} \Leftrightarrow \frac{dy}{dx} = x e^{x^2} \Leftrightarrow dy = x e^{x^2} dx \Leftrightarrow \int dy = \int x e^{x^2} dx \Leftrightarrow y = \int x e^{x^2} dx$$

$$y = \int x e^{x^2} dx$$

$$u = x^2 \Rightarrow du = 2x$$

$$\therefore \int x e^{x^2} dx = \frac{1}{2} \int e^u du = \frac{1}{2} e^{x^2} + C = y$$

$$(33) \quad \frac{dy}{dt} = \frac{t^2}{\sqrt{2+3t}} \Leftrightarrow \int dy = \int \frac{t^2}{\sqrt{2+3t}} dt \Leftrightarrow y = \int \frac{t^2}{\sqrt{2+3t}} dt$$

$$u = t^2 \quad v = \frac{1}{3} (2+3t)^{(1/2)}$$

$$du = 2t dt \quad dv = \frac{1}{(2+3t)^{(1/2)}}$$

$$\int \frac{t^2}{\sqrt{2+3t}} dt = \frac{1}{3} t^2 (2+3t)^{(1/2)} - \frac{2}{3} \int t (2+3t)^{(1/2)} dt$$

$$u = t \quad v = -\frac{4}{9} (2+3t)^{(3/2)}$$

$$du = dt \quad dv = -\frac{2}{3} (2+3t)^{(1/2)} dt$$