(45) $\int x^2 e^{2x} dx$ (using tabular integration - this is the preferred method to do integration)

Let $u = x^2$ $\quad dv = e^{2x}$

$-u' = -2x \quad \int dv = \frac{1}{2}e^{2x}$ (don't forget to add a negative sign)

$u'' = 2 \quad \int \left\{\int dv\right\} dv = \frac{1}{4}e^{2x}$ (Note that the negative sign does not travel through)

$u''' = 0 \quad \int \left\{\int \left\{\int dv\right\} dv\right\} dv = \frac{1}{8}e^{2x}$

Then multiply down diagonally, as follows

$$\int x^2 e^{2x} dx = x^2\left(\frac{1}{2}e^{2x}\right) + \frac{1}{4}e^{2x}(-2x) + 2\frac{1}{8}e^{2x} + C = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

Note : In the future we will define $\int dv = F_1(x)$, $\int F_1(x)dx = F_2(x)$ and so on. This makes the pattern of tabular integration as follows : $uF_1(x) + (-u')F_2(x) + u''F_3(x) + \cdots$

(47) $\int x^3 \sin x dx$

Let $u = x^3$ $\quad dv = \sin x dx$

$u' = -3x^2 \quad F_1(x) = -\cos x$

$u'' = 6x \quad F_2(x) = -\sin x$

$u^{(3)} = -6 \quad F_3(x) = \cos x$

$u^{(4)} = 0 \quad F_4(x) = \sin x$

$\therefore \int x^3 \sin x dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6\sin x + C$

(49) $\int x \sec^2 x dx$

Let $u = x$ $\quad dv = \sec^2 x dx$

$u' = -1 \quad F_1(x) = \tan x$

$u'' = 0 \quad F_2(x) = -\ln|\cos x|$

$\therefore \int x \sec^2 x dx = x \tan x + \ln|\cos x| + C$