

$$(61) \quad \int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

Proof

$$\begin{aligned} \text{Let } u &= x^n & v &= -\cos x \\ du &= nx^{n-1} dx & dv &= \sin x dx \end{aligned}$$

$$\therefore \int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

Q.E.D.

$$(63) \quad \int x^n \ln x dx = \frac{x^{n+1}}{(n+1)^2} [(n+1) \ln x - 1] + C$$

Proof

$$\text{Let } u = \ln x \quad v = \frac{1}{(n+1)} x^{n+1}$$

$$du = \frac{1}{x} \quad dv = x^n dx$$

$$\therefore \int x^n \ln x dx = \frac{1}{(n+1)} x^{n+1} \ln x - \frac{1}{(n+1)} \int x^n dx$$

$$- \frac{1}{(n+1)} \int x^n dx = - \frac{1}{(n+1)} \left\{ \frac{1}{(n+1)} x^{n+1} \right\} + C = - \frac{1}{(n+1)^2} x^{n+1} + C$$

$$\therefore \int x^n \ln x dx = \frac{1}{(n+1)} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C = \frac{x^{n+1}}{(n+1)^2} [(n+1) \ln x - 1] + C$$

Q.E.D.