$$\int e^{ax}\sin bx dx = \frac{e^{ax}(a\sin bx - bx)}{a^2+b^2} + C$$

Proof

$$\int e^{ax}\sin bx dx = \frac{1}{a}e^{ax}\sin bx - \frac{b}{a}\int e^{ax}\cos bx dx$$

Let $u = \sin bx$ $\qquad v = \frac{1}{a}e^{ax}$

$du = b\cos bx dx \qquad dv = e^{ax}dx$

$$-\frac{b}{a}\int e^{ax}\cos bx dx = -\frac{b}{a}\left\{\frac{1}{a}e^{ax}\cos bx + \frac{b}{a}\int e^{ax}\sin bx dx\right\} = -\frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}\int e^{ax}\sin bx dx$$

Let $u = \cos bx$ $\qquad v = \frac{1}{a}e^{ax}$

$du = -b\sin bx dx \qquad dv = e^{ax}dx$

$$\therefore \int e^{ax}\sin bx dx = \frac{1}{a}e^{ax}\sin bx - \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}\int e^{ax}\sin bx dx$$

$$\Leftrightarrow \int e^{ax}\sin bx dx + \frac{b^2}{a^2}\int e^{ax}\sin bx dx = \frac{1}{a}e^{ax}\sin bx - \frac{b}{a^2}e^{ax}\cos bx$$

$$\Leftrightarrow \left(1+\frac{b^2}{a^2}\right)\int e^{ax}\sin bx dx = \frac{1}{a}e^{ax}\sin bx - \frac{b}{a^2}e^{ax}\cos bx$$

$$\Leftrightarrow \frac{1}{a^2}\left(a^2+b^2\right)\int e^{ax}\sin bx dx = \frac{1}{a^2}e^{ax}\left(a\sin bx - b\cos bx\right)$$

$$\Leftrightarrow \left(a^2+b^2\right)\int e^{ax}\sin bx dx = e^{ax}\left(a\sin bx - b\cos bx\right)$$

$$\Leftrightarrow \int e^{ax}\sin bx dx = \frac{e^{ax}\left(a\sin bx - b\cos bx\right)}{\left(a^2+b^2\right)} + C$$

Q.E.D.