$$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta}) \quad and \qquad \cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

Proof

By Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad and \qquad e^{-i\theta} = \cos\theta - i\sin\theta$$

Using Cramer's Rule

$$\det\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2, \quad \det_y\begin{vmatrix} 1 & e^{i\theta} \\ 1 & e^{-i\theta} \end{vmatrix} = -(e^{i\theta} - e^{-i\theta}), \quad \det_x\begin{vmatrix} e^{i\theta} & 1 \\ e^{-i\theta} & -1 \end{vmatrix} = -(e^{i\theta} + e^{-i\theta})$$

$$\therefore\ \cos\theta = \frac{\det_x}{\det A} = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad and$$

$$i\sin\theta = \frac{\det_y}{\det A} = \frac{e^{i\theta} - e^{-i\theta}}{2} \Leftrightarrow \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

Q.E.D.

For matrices A and B on which multiplication is defined.

$$AB = BA \Rightarrow (A+B)^2 = A^2 + 2AB + B^2$$

Proof

By a calculation

$$(A+B)^2 = (A+B)(A+B) = A^2 + AB + BA + B^2.$$

$$Since \ \ AB = BA \ \ (that\ is,\ A\ commutes\ with\ B)$$

$$(A+B)^2 = A^2 + 2AB + B^2 = A^2 + 2BA + B^2$$

Q.E.D.

$$(AB)^T = B^T A^T$$

Proof

$$Let \qquad A = [a_{ij}]_{m\times p} \qquad and \qquad B = [b_{ij}]_{p\times n}$$

Therefore the product of A and B is defined. Moreover the ij^{th} entry of AB is

$$AB = \left[\sum_{k=1}^{p} a_{ik} b_{kj}\right]_{m\times n}$$

The transpose of AB,

$$(AB)^T = \left[\sum_{k=1}^{p} a_{jk} b_{ki}\right]_{n\times m}$$

The transpose of A and B are

$$B^T = [b_{ij}]_{n\times p} \ \ and \ \ A^T = [a_{ji}]_{p\times m}$$

Therefore the product of B^T and A^T is defined. Moreover the ij^{th} entry of $B^T A^T$ is

$$B^T A^T = \left[\sum_{k=1}^{p} b_{ki} a_{jk}\right]_{n\times m} \qquad Since\ b_{ki}, a_{jk} \in \Re$$

$$B^T A^T = \left[\sum_{k=1}^{p} a_{jk} b_{ki}\right]_{n\times m}$$

$\therefore \qquad (AB)^T = B^T A^T .$

Q.E.D.

For all $n \in N$ with $n \geq 4,\ n! > 2^n$

Proof (by mathematical induction)

$Let\ T = \{n \in N : n! > 2^n\}$

$Claim 1:\ 4 \in T$

$if\ n = 4,\ then\ n! = 24$

$if\ n = 4,\ then\ 2^n = 16$

$\therefore 4! = 24 > 16 = 2^4$

$\therefore 4 \in T$

$Claim 2:\ if\ k \in T,\ then\ (k+1) \in T$

$(k+1)! = (k+1)k!$

$Since\ k! > 2^k,\ it\ follows\ that$

$(k+1)k! > (k+1)2^k$

$Since\ k \geq 4,\ it\ follows\ that\ k+1 \geq 5 > 2$

$\therefore (k+1)k! > (k+1)2^k > 2 \cdot 2^k = 2^{k+1}$

Summary: It follows from Claim1 and Claim2 that T={4,5,6,…}

In other words, $n! > 2^n\ \forall\ n \in N \ni n \geq 4$