Statistical Mechanics of Cross-Modal Neural Plasticity

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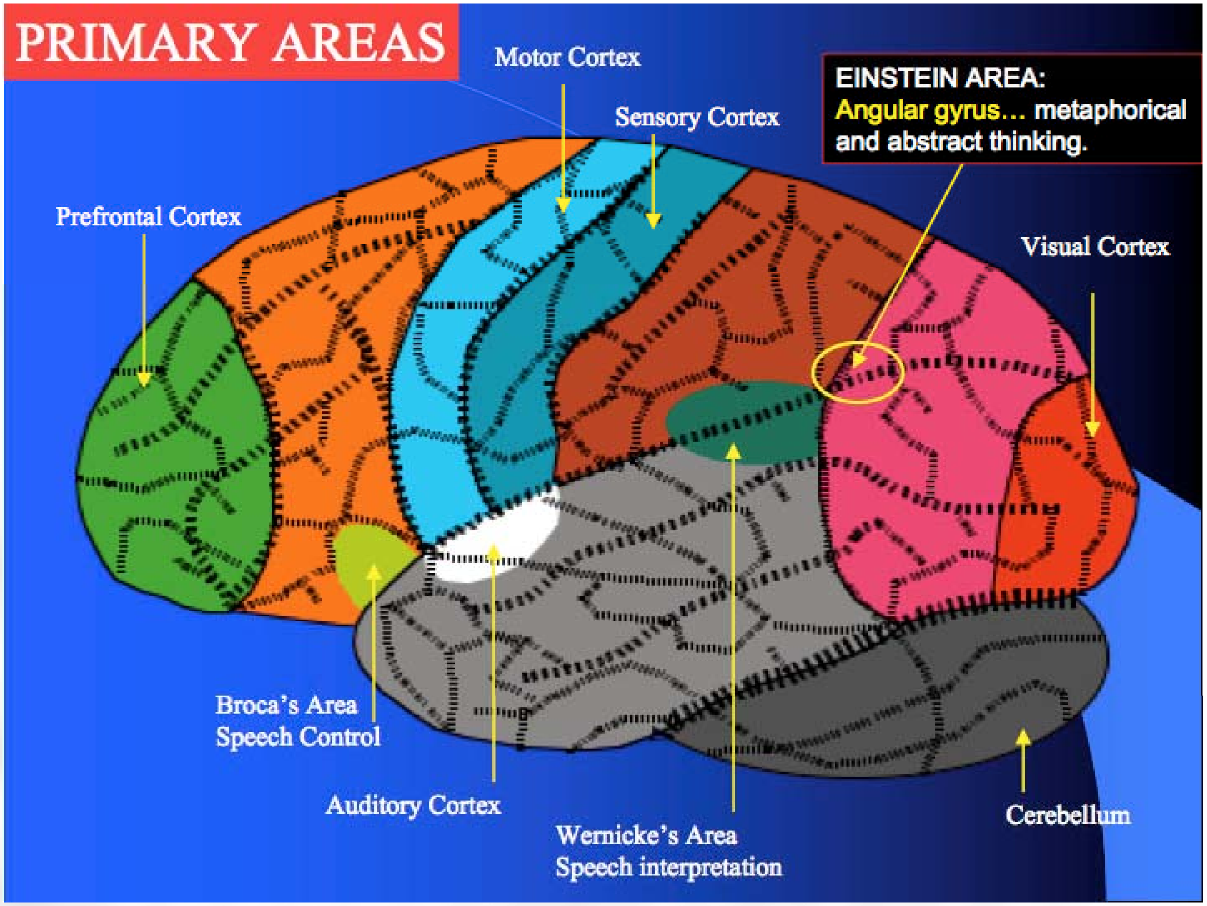
Introduction: Phenomena Observed (1/2)

In early cognitive science it was thought that different sensory modules (for vision, hearing, touch, smell and taste) function independent of each other.



Introduction: Phenomena Observed (2/2)

Recently, it has been argued that **cross-modal interactions** are present

And the cortical pathways previously thought to be sensory-specific are modulated by signals from other modalities

Some influential experimental studies supporting this hypothesis are:

- vision and auditory system:
  - E.g. McDonald(2000): Hearing by eye
- sound and motion systems:
  - Soto-faraco et al. (2004)

Brain imaging and recording studies

- using MEG (magnetoencephalography):
  - Soto-faraco et al. (2004)
- fMRI (functional Magnetic Resonance Imaging):
  - Kaiser et al. (2005): Hearing Lips.

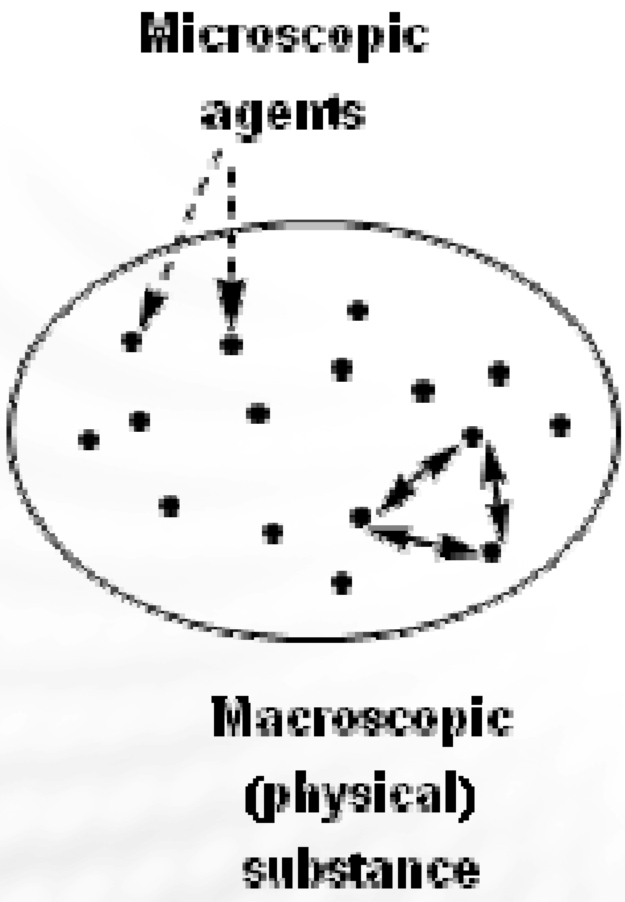
Research problem

Finding a mathematical model for describing this phenomena.

- There is no evidence of existence of any mathematical model for describing this multi-modal behavior of the brain.
- It would be very useful to recognize the statistical aspects of this system mathematically

Statistical Mechanics

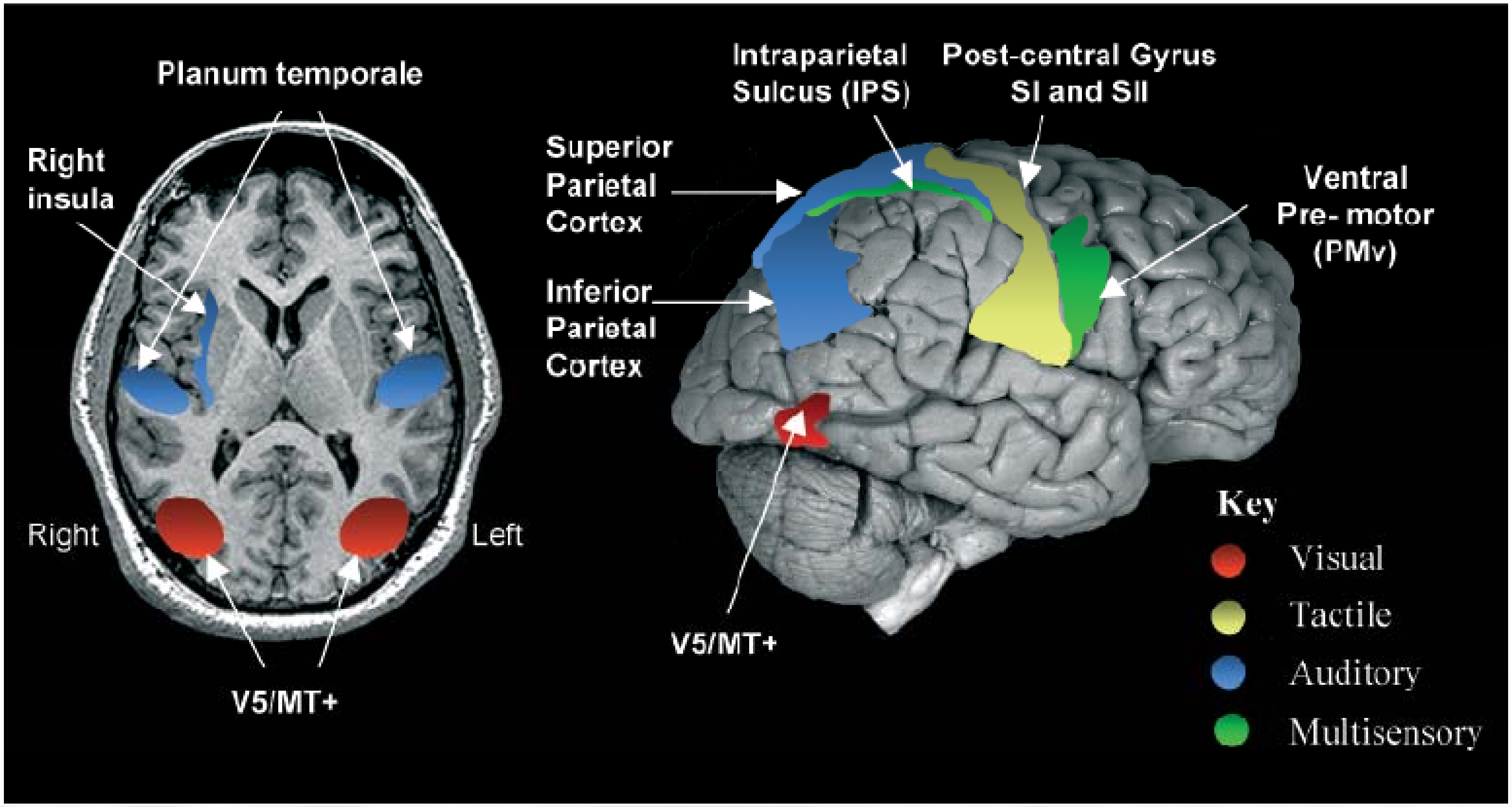
- Statistical Mechanics
  - enables to derive macroscopic properties of a substance from a probability distribution that describes the complicated interactions among the individual constituent particles.
- The mathematical language
  - of SM (we use here) is the large deviations.



Terminology (2/3)

Crossmodal

Crossmodal perception hypothesis that sensory processing within a single sense is modulated by information in and attention towards the other senses.



©S. Soto-faraco (2004)

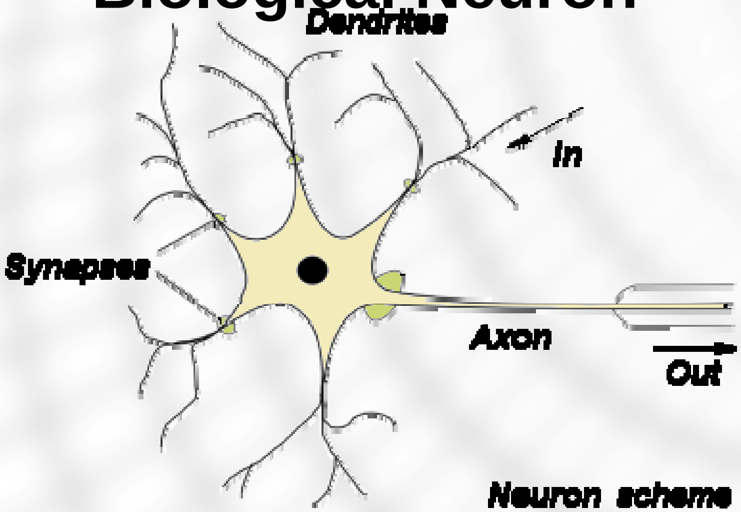
Terminology (3/3)

Neural Plasticity

In neuroscience, synaptic plasticity is the ability of the connection, or synapse, between two neurons to change in strength.

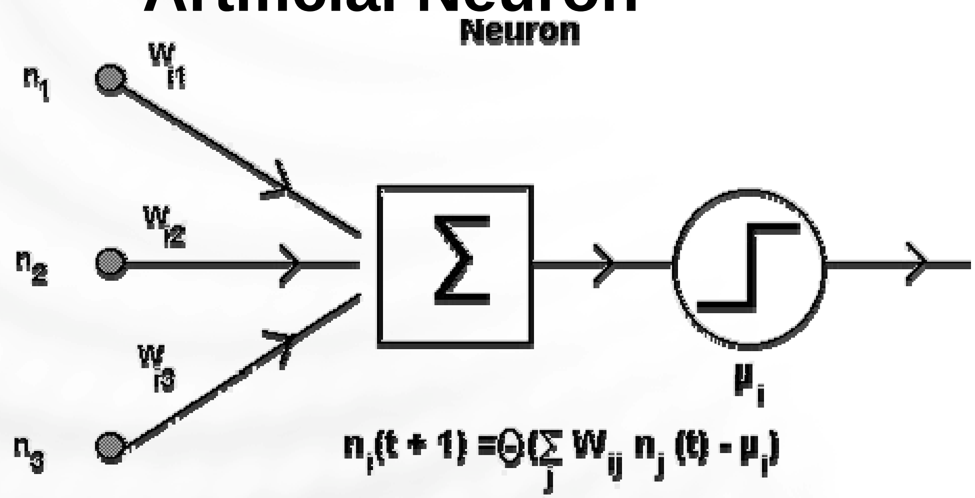
So connection strength adjust according to the inputs they received.

Biological Neuron



Activates if input exceeds a certain threshold

Artificial Neuron



Activates if the weighted sum of the inputs exceeds a certain threshold

Justification: Applications

This insight can be applied to improve machine learning:

- visual data backed by auditory clues which incorporates natural ability of humans to reduce audio ambiguity using visual cues,
- speech recognition (sounds with lip movements) to gain better understanding of the surrounding from noisy signals that are coming from multiple sensories

To make the learning process more effective

- majority of knowledge held by adults (75%) is learned through seeing; hearing is the next most effective (about 13%)
- and the other senses - touch, smell and taste account for 12% of what we know.
- By stimulating the senses, learning can be enhanced

To enhance the differently-abled people.

- E.g. The vOICe vision technology for the totally blind

Modeling Cortex (1/4)

Brain

- The brain is the center of the nervous system in most animals.
- Consists of 50–100 billion (10^{11}) interconnected neurons
- With orders of ~100 trillion (10^{14}) connections
- Each collectively works
- Communication via chemical process

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Modeling Cortex (2/4)

Bit of History…

To model the dynamical activity in the cortex now there are networks with up to 10^5 neurons.

Basically they focus on the neuron activity and stability with regards to one type of external stimuli.

Mountcastle (1957) discovered the columnar organization of the brain - all neurons in a cortical column - neocortical column (NCC) - functions identically

So we can now concentrate on the undergoing activities in columnar level rather than on the entire brain

e.g. Blue Brain Project (BBP).

Douglas & Martin (2004) suggest a simple model of cortical processing that is consistent with the major features of cortical circuits

Modeling Cortex (3/4)

Assumptions…

The brain is a complex, and nonlinear system and brain state variables exhibit high dimensional chaos

The neuron interactions are mostly happening among nearest-neighbors,

This also implies that collective aspects of this system may be studied similarly to other collective systems

and mathematically model the activities pertaining to neo columns.

Cortex functions as an associative memory

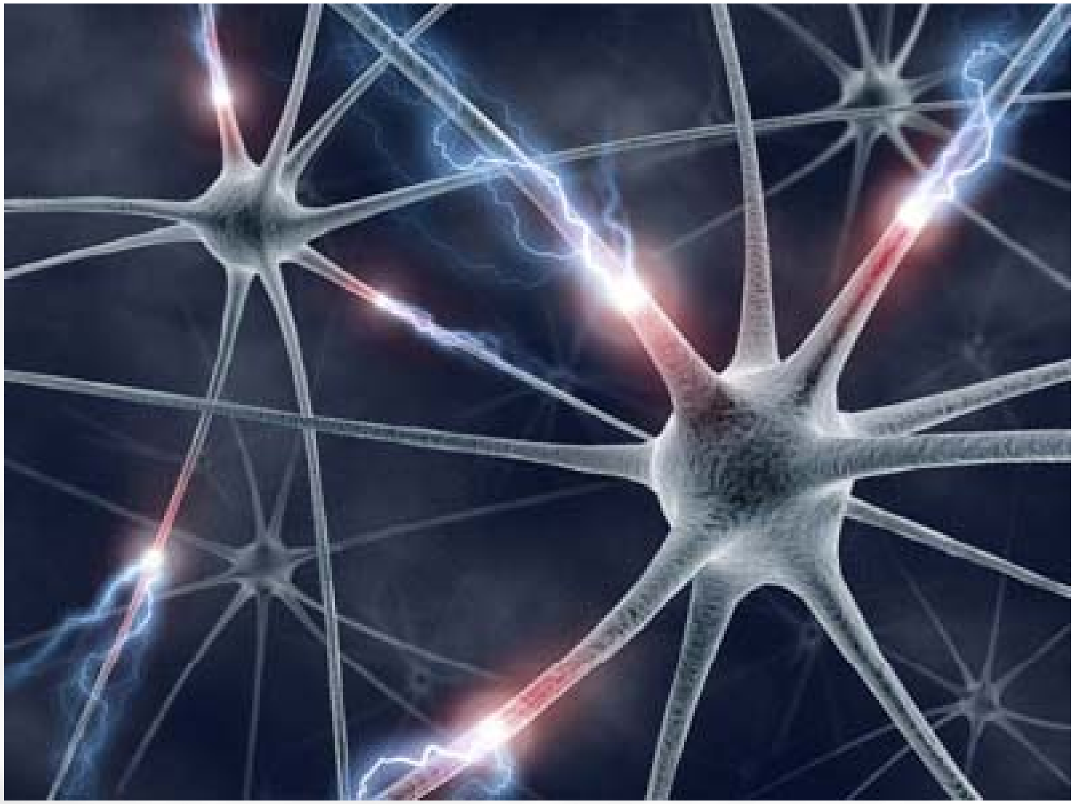
Ingber's series of papers (1982-92) addresses developing statistical mechanics of neocortical interactions (SMNI) which closely mimics the neocortex

Modeling Cortex (4/4)

Neural Network

Brain is comprised of a highly interconnected network of large number of units called neurons

which can be abstractly considered as a simple mathematical function of the weighted sum of the input signals.



Question

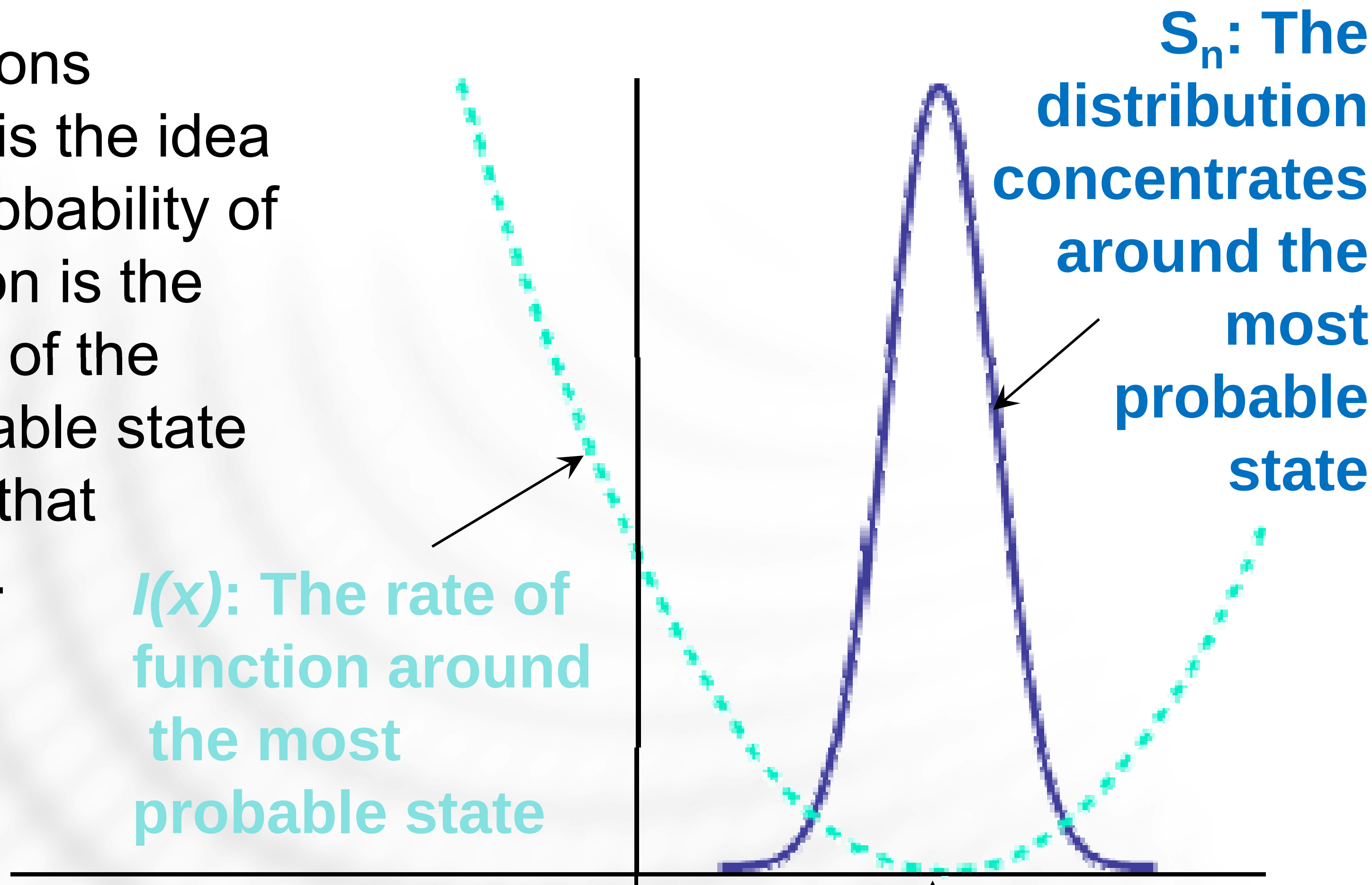
What happens in the multi-sensory area when two correlated inputs from different sensory modules are present?

- There is very little done on rigorous mathematical (rather via mathematical statistical mechanics) modeling aspects on multi-modalities though there had been extensive and excellent investigations done on single-modality aspects.
- We can find distribution functions at one scale and integrating to form more macroscopic variables. Our concern is to provide some useful enhancement to such a complex model.

Techniques Used (1/3)

Large Deviations Techniques (LDT)

Large deviations technique is the idea that the probability of a fluctuation is the probability of the most probable state leading to that fluctuation.



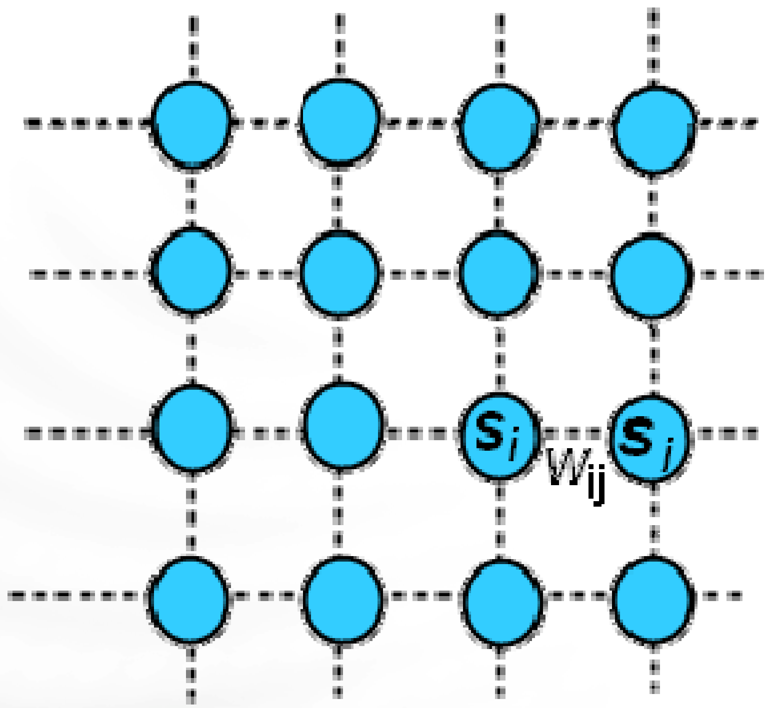
$$P(S_n > x) \approx \exp[-n\, I(x)]$$

The most probable state

Techniques Used (2/3)

Ising Model

- A single network consisting of number of neurons acting identical to each other can be approximated by an Ising model
- Here the neurons are arranged in a lattice structure
- Each neuron is either on or off (±1) state s_i and
- Influences by other neurons with random couplings w_{ij}
- Interactions are among only the nearest-neighbors.



Energy for the entire network

$$H := \sum w_{ij} . s_i . s_j$$

Techniques Used (3/3)

Monte-Carlo Simulation

- Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling to compute their results.
- Often used when simulating physical and mathematical systems.
- Rely on repeated computation and random or pseudo-random numbers
- Most suited to calculation by a computer.
- Tend to be used when it is unfeasible or impossible to compute an exact result with a deterministic algorithm

Proposed Modeling Approaches (1/6)

We purpose abstract models in which the interactions coming from different sensories are treated as interaction between two interconnecting networks (with their own interactions).

Assuming each neuron is in either on or off (± 1) state and influences by other neurons with random couplings.

The nature of these couplings and topology of the lattice gives rise to different dimension of the problem (Scenarios A, B and C)

Proposed Modeling Approaches (2/6)

(A) Interacting two nearest neighbor Ising spin systems

Possible model Hamiltonian for the problem is

$$\mathcal{H} := \frac{1}{N^2}\sum_{(i,j)} W_{ij}\sigma_i\sigma_j + \frac{1}{M^2}\sum_{(i,j)} V_{ij}\nu_i\nu_j + \frac{1}{N}\sum_{i=1}^{N}\sigma_i h_i + \frac{1}{M}\sum_{i=1}^{M}\nu_i g_i,$$

where *N* and *M* are the number of spins in the two populations.

The first two summation are over nearest neighbors while the third term quantifies the effect on the *N* spins of the first population by the second and the fourth term - the opposite effect.

Here, W_{ij}, V_{ij}, g_i, and h_i all shall have a-priori probability distributions where W_{ij}, V_{ij}, denote the interactions among the i^{th} and j^{th} neurons in the two systems respectively.

Proposed Modeling Approaches (3/6)

(B) Interacting two mean-field Ising spin systems (or Hopfield model).

Here we consider two interacting Hopfield model systems and propose the following model Hamiltonian:

$$\mathcal{H} := \frac{1}{N^2}\sum_{ij} W_{ij}\sigma_i\sigma_j + \frac{1}{M^2}\sum_{ij} V_{ij}\nu_i\nu_j + \frac{1}{N}\sum_{i}\sigma_i h_i + \frac{1}{M}\sum_{j}\nu_j g_j$$

where $W_{ij} = (1/N)\sum_{\mu}\xi_i^{\mu}\xi_j^{\mu}$

ξ_i, ξ_j, h_i and g_j all have a priori probability distributions where ξ_i is the interaction strength.

Proposed Modeling Approaches (4/6)

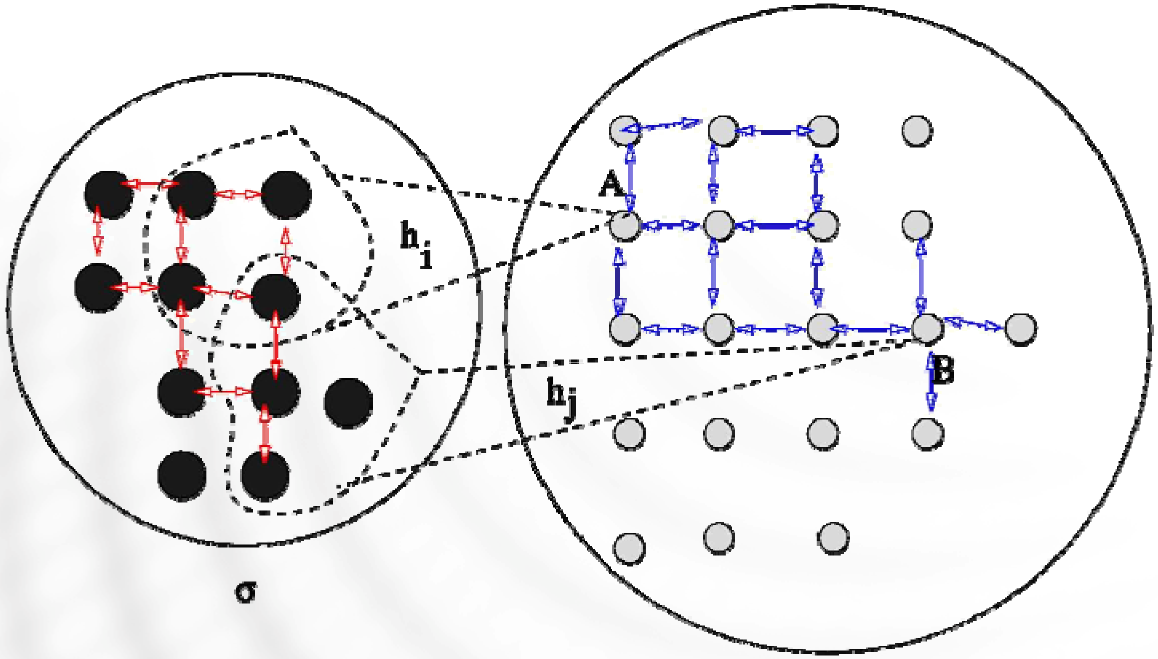
(C) Interacting two directed polymers in a random media.

$$\mathcal{H} := \frac{1}{N^2}\sum_i W_i + \frac{1}{M^2}\sum_i V_i + \frac{1}{N}\sum_i^N \sigma_i h_i + \frac{1}{M}\sum_j^M v_j g_j + \frac{1}{N_1 M_1}\left(\sum_{i=1}^{N_1}\frac{1}{\sqrt[p_i]{N}}\prod_{j=1}^{p_i}\sigma_j\right)\left(\sum_{i=1}^{M_1}\frac{1}{\sqrt[q_i]{M}}\prod_{j=1}^{q_i}v_j\right)$$

where

$$N_1 = \sum\binom{N}{p_i}, M_1 = \sum\binom{M}{q_i}$$

Proposed Modeling Approaches (5/6)



Two interacting binary-neural (Ising spin) populations on two-dimensional square lattices with nearest neighbor interactions, whilst the inter-network topology is defined for every *p*-group of spins between the two systems.

Proposed Modeling Approaches (6/6)

Free energy estimates

Every system seeks to achieve a minimum of free energy

We investigate the equilibria of the models (as defined in A, B and C) via analytical solutions to the free-energy $f(\beta)$, which shall be computed via evaluating the limit

$$-\beta f(\beta) := \lim_{n\to\infty} \frac{1}{n} \log \mathcal{Z}_n(\beta)$$

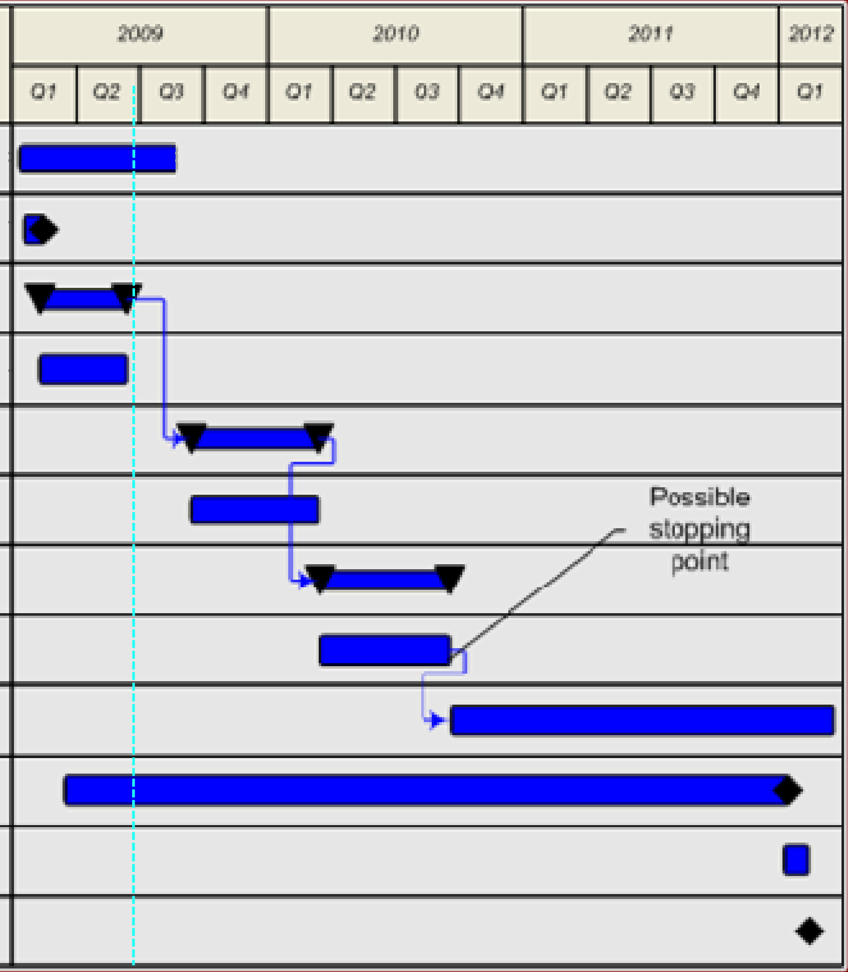
where $$\mathcal{Z}_n(\beta) := \sum \exp\{-\beta \mathcal{H}_n\},$$

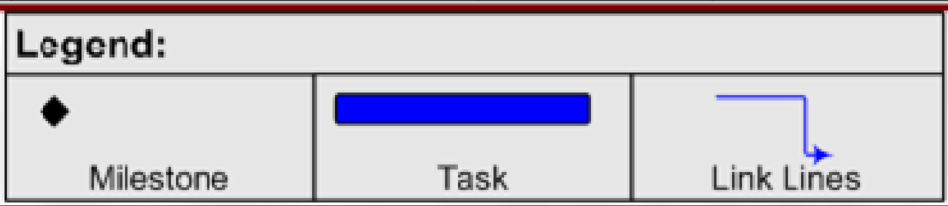
$\mathcal{H}$ the Hamiltonian of the system and β is defined as the inverse temperature and *n* is the number of particles in the system.

Verify the analytical solution using Monte-Carlo simulations.

Time Schedule

ID	Task Name	Duration
1	Background Reading	32w
2	Preparing Project Proposal	4w
3	**Model A:**	**18w**
4	Developing theory and Simulations	18w
5	**Model B:**	**26w**
6	Developing theory and Simulations	26w
7	**Model C:**	**27w**
8	Developing theory and Simulations	27w
9	Possible Extensions	78w
10	Working on Project Thesis	148w
11	Formatting, Drawing, etc	5w
12	Final Project Report	0w





*Schedule as at 7/15/2009

Q & A

Thank You!