

1. COMMON FACTORING:

$$\begin{aligned} \text{(a)} \quad 3x - 6 \\ = 3(x - 2) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 20x^3 + 4x \\ = 4x(5x^2 + 1) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 35x^3y^7 - 14xy^5 + 42x^2y^3 \\ = 7xy^3(5x^2y^4 - 2y^2 + 6x) \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 5(a+3)^2 + 4(a+3) \\ = (a+3)(5(a+3) + 4) \\ = (a+3)(5a + 19) \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 2(y-5)^2 - y + 5 \\ = 2(y-5)^2 - (y-5) \\ = (y-5)(2(y-5) - 1) \\ = (y-5)(2y - 11) \end{aligned}$$

2. SIMPLE TRINOMIAL: $x^2 + bx + c$

$$\begin{aligned} \text{(a)} \quad x^2 + 2x - 15 \\ = (x+5)(x-3) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad t^2 - 10t + 21 \\ = (t-7)(t-3) \end{aligned}$$

3. GENERAL TRINOMIAL: $ax^2 + bx + c$

$$\text{(a)} \quad 2x^2 + 3x - 5$$

trial + error

$$\begin{array}{r|rr} 2x & x & 5 & -5 \\ x & -2x & -1 & 1 \end{array}$$

$$= (2x+5)(x-1)$$

decomposition

$$a \cdot c = 2(-5) = -10$$

$$b = 3$$

$\therefore$ Need to find two #s that multiply to -10 but add to 3. $\Rightarrow -2 \& 5$

$$= 2x^2 - 2x + 5x - 5$$

$$= 2x(x-1) + 5(x-1)$$

$$= (x-1)(2x+5)$$

$$\text{(b)} \quad 6d^2 + 19d + 15$$

trial + error

$$\begin{array}{r|rr} 6d & d & 2d & 3d & 3 & 1 \\ d & 6d & 3d & 2d & 5 & 15 \end{array}$$

$$= (2d+3)(3d+5)$$

decomposition

$$\begin{cases} a \cdot c = 90 \\ b = 19 \end{cases}$$

$$\rightarrow \boxed{9} \cdot \boxed{10} = 90$$

$$\boxed{9} + \boxed{10} = 19$$

$$= 6d^2 + 9d + 10d + 15$$

$$= 3d(2d+3) + 5(2d+3)$$

$$= (2d+3)(3d+5)$$

4. PERFECT SQUARE TRINOMIAL: $A^2 \pm 2AB + B^2 = (a+b)^2$

$$\begin{aligned} \text{(a)} \quad m^2 + 8m + 16 & \quad a = m \\ & \quad b = 4 \\ & = (m+4)^2 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 4p^2 - 20pq + 25q^2 & \quad a = 2p \\ & \quad b = -5q \\ & = (2p - 5q)^2 \end{aligned}$$

5. DIFFERENCE OF SQUARES: $A^2 - B^2 = (a+b)(a-b)$

$$\begin{aligned} \text{(a)} \quad 25w^2 - 81v^2 & \quad a = 5w \\ & \quad b = 9v \\ & = (5w - 9v)(5w + 9v) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 100g^2 - 1 & \quad a = 10g \\ & \quad b = 1 \\ & = (10g + 1)(10g - 1) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (x+3)^2 - (2x+1)^2 & \quad a = x+3 \\ & \quad b = 2x+1 \\ & = (x+3+2x+1)(x+3-(2x+1)) \\ & = (3x+4)(-x+2) \end{aligned}$$

6. COMBINATIONS: two or more types of factoring may be involved

$$\begin{aligned} \text{(a)} \quad 4x^2 + 12x - 16 & \\ & = 4(x^2 + 3x - 4) \\ & = 4(x+4)(x-1) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 32b^2 - 98 & \\ & = 2(16b^2 - 49) \\ & = 2(4b-7)(4b+7) \end{aligned}$$

ALSO: Check that none of the factors can be factored further.

Verify your answer by expanding and comparing to the original question.

Not every expression is factorable.

$$1) 4x + 6x^2 \\ = 2x(2 + 3x)$$

$$2) 5x^3 + 25x^2 + 30x \\ = 5x(x^2 + 5x + 6) \\ = 5x(x+2)(x+3)$$

$$3) 12ax + 3ay + 15az \\ = 3a(4x + y + 5z)$$

$$4) x^2 + 3x + 2 \\ = (x+1)(x+2)$$

$$5) y^2 + y - 30 \\ = (y+6)(y-5)$$

$$6) a^2 - 17a + 60 \\ = (a-20)(a+3)$$

$$7) 2x^2 - 6x + 4 \\ = 2(x^2 - 3x + 2) \\ = 2(x-2)(x-1)$$

$$8) 3k^2 + 15k + 18 \\ = 3(k^2 + 5k + 6) \\ = 3(k+2)(k+3)$$

$$9) 5x^2 + 5x - 30 \\ = 5(x^2 + x - 6) \\ = 5(x+3)(x-2)$$

$$10) 3x^2 - 7x + 4 \\ = 3x^2 - 3x - 4x + 4 \\ = 3x(x-1) - 4(x-1) \\ = (x-1)(3x-4)$$

$$11) 2x^2 + 9x + 10 \\ = 2x^2 + 4x + 5x + 10 \\ = 2x(x+2) + 5(x+2) \\ = (x+2)(2x+5)$$

$$12) 2b^2 - b - 15 \\ = 2b^2 - 6b + 5b - 15 \\ = 2b(b-3) + 5(b-3) \\ = (b-3)(2b+5)$$

$$13) 4c^2 - 8c - 21 \\ = 4c^2 + 6c - 14c - 21 \\ = 2c(2c+3) - 7(2c+3) \\ = (2c+3)(2c-7)$$

$$14) 4x^2 - 17x + 15 \\ = 4x^2 - 12x - 5x + 15 \\ = 4x(x-3) - 5(x-3) \\ = (x-3)(4x-5)$$

$$15) 5w^2 + 11w + 2 \\ = 5w^2 + 10w + w + 2 \\ = 5w(w+2) + 1(w+2) \\ = (w+2)(5w+1)$$

$$16) 36x^2 - 48x - 20 \\ = 4(9x^2 - 12x - 5) \\ = 4(9x^2 + 3x - 15x - 5) \\ = 4[3x(3x+1) - 5(3x+1)] \\ = 4(3x+1)(3x-5)$$

$$17) 48x^2 - 200x + 200 \\ = 8(6x^2 - 25x + 25) \\ = 8(6x^2 - 15x - 10x + 25) \\ = 8[3x(2x-5) - 5(2x-5)] \\ = 8(2x-5)(3x-5)$$

$$18) 24n^3 + 68n^2 + 48n \\ = 4n(6n^2 + 17n + 12) \\ = 4n(6n^2 + 9n + 8n + 12) \\ = 4n[3n(2n+3) + 4(2n+3)] \\ = 4n(2n+3)(3n+4)$$

$$19) am - an + bm - bn$$

$$= a(m-n) + b(m-n)$$

$$= (m-n)(a+b)$$

$$20) 10ax + 4ay - 15x - 6y$$

$$= 10ax - 15x + 4ay - 6y$$

$$= 5x(2a-3) + 2y(2a-3)$$

$$= (2a-3)(5x+2y)$$

$$21) 9am + 3bm + 6an + 2bn$$

$$= 3m(3a+b) + 2n(3a+b)$$

$$= (3m+2n)(3a+b)$$

$$22) x^3 + x^2 + x + 1$$

$$= x^2(x+1) + 1(x+1)$$

$$= (x+1)(x^2+1)$$

$$23) 1 + ab + a + b$$

$$= ab + a + b + 1$$

$$= a(b+1) + 1(b+1)$$

$$= (b+1)(a+1)$$

$$24) (x+y)^2 - x - y$$

$$= (x+y)^2 - (x+y)$$

$$= (x+y)((x+y)-1)$$

$$= (x+y)(x+y-1)$$

$$25) 9x^2 - 30xy + 25y^2$$

$$= (3x-5y)^2$$

$$26) 16a^2 + 24ab + 9b^2$$

$$= (4a+3b)^2$$

$$27) 50a^2 - 120ab + 72b^2$$

$$= 2(25a^2 - 60ab + 36b^2)$$

$$= 2(5a-6b)^2$$

$$28) x^2 - y^2$$

$$= (x+y)(x-y)$$

$$29) 49x^2 - 64y^2$$

$$= (7x+8y)(7x-8y)$$

$$30) 18a^3 - 50ab^2$$

$$= 2a(9a^2 - 25b^2)$$

$$= 2a(3a-5b)(3a+5b)$$

$$31) (2x-1)^2 - y^2$$

$$= (2x-1+y)(2x-1-y)$$

$$32) 9k^2 - (5m-1)^2$$

$$= (3k-(5m-1))(3k+(5m-1))$$

$$= (3k-5m+1)(3k+5m-1)$$

$$33) 4a^2 - 9 - 2ab + 3b$$

$$= (2a+3)(2a-3) - b(2a-3)$$

$$= (2a-3)(2a+3-b)$$

$$34) (5x)^2 + 7(5x) + 12$$

$$= (5x+3)(5x+4)$$

$$35) (x^2+3x)^2 - 2(x^2+3x) - 8$$

$$= (x^2+3x-4)(x^2+3x+2)$$

$$= (x+4)(x-1)(x+1)(x+2)$$

$$36) 6(a+b)^2 + 17(a+b) + 5$$

$$let (a+b) = x$$

$$= 6x^2 + 17x + 5$$

$$= 6x^2 + 2x + 15x + 5$$

$$= 2x(3x+1) + 5(3x+1)$$

$$= (3x+1)(2x+5)$$

$$= (3a+3b+1)(2a+2b+5)$$

$$37) m^2 - 9 + 6n - n^2$$

$$= m^2 - (n^2 - 6n + 9)$$

$$= m^2 - (n-3)^2$$

$$= (m-n+3)(m+n-3)$$

$$38) 49 - (2y-w)^2$$

$$= (7-2y+w)(7+2y-w)$$

$$39) (x+y)^2 - (a+b)^2$$

$$= (x+y-a-b)(x+y+a+b)$$

$$40) a^2 + 2ab + b^2 - c^2 + 6c - 9$$

$$= (a+b)^2 - (c-3)^2$$

$$= (a+b+c-3)(a+b-c+3)$$