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ON THE EFFECT OF THE MATERIAL VISCOSITY AND ELASTIC FOUNDATIONS OF VARIABLE MODULUS ON THE DYNAMIC STABILITY OF CANTILEVERED PIPES

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1. Introduction

The dynamic stability of fluid-conveying pipes has been extensively studied in the past 40 years (see, e.g., the comprehensive book by Païdoussis [1]). In general, it has been established that if an initially straight pipe conveys inviscid fluid with a relatively low velocity, then each disturbance applied to that pipe causes vibration diminishing with the time. In this case, the initial equilibrium state of the pipe is referred to as a stable one. However, for fluid velocities higher than a certain value (called critical flow velocity) even small disturbances could result in vibration with larger and larger amplitudes. Under these circumstances, the pipe equilibrium state is referred to as an unstable one.

Usually the pipes are supported at the ends but, for different reasons, they are often supported along the span too. From mathematical point of view, these internal supports could be described as a continuous foundation the pipe is resting on. Surprisingly, in spite of the intuitive expectation, it turns out that a foundation does not always stabilize a pipe. The same holds true with respect to the internal damping as well.

In 1978, Becker, Haugher and Winzen [2] considered the dynamic stability of cantilevered viscoelastic pipes on foundations of constant modulus for several small mass ratios. Later, Lottati and Kornecki [3] studied the same problem but for all admissible values of the mass ratio and several different values of the internal damping coefficient. In these works, it has been established that Winkler foundations of constant modulus have a stabilizing effect, as expected (see also paper [4] by Doared and de Langre). However, the internal damping has been found to have either destabilizing or stabilizing effect on the pipe depending on the mass ratio. Elishakoff and Impolonia [5] and Djondjorov [6] have studied the dynamic stability of cantilevered pipes on foundations of constant modulus that support only a part of the pipe span. They have obtained that such foundations could either destabilize or stabilize the pipe depending on the position and length of the foundations. Djondjorov, Vassilev and Dzhupanov [7] and Djondjorov [8] have examined cantilevered pipes on Winkler foundations whose modulus is a certain sixth-, second- or first-order polynomial. They have concluded that all such foundations stabilize the pipe. Vassilev and Djondjorov [9] considered elastic cantilevered pipes on foundations whose modulus is a second-order polynomial vanishing at the pipe ends and found that such foundations destabilize the pipes.

The aim of the present note is to analyse the effect of the internal damping on the dynamic stability of cantilevered viscoelastic pipes lying on the elastic foundations of Winkler type considered in [9]. The computational procedure used here is the one developed in [9] but modified to account for the internal damping. Using an appropriate Green function, a Volterra integral equation equivalent to the considered governing differential equation is derived. It is solved by the Neumann series method in Maple environment and is used to verify whether a frequency, obtained by the Galerkin computational procedure at given pipe parameters (flow velocity, mass ratio, internal damping and foundation parameters), corresponds to a sufficiently good approximate solution to the respective two-point boundary value problem.

2. Governing Equations and Boundary Conditions

The small transverse vibration of an initially straight viscoelastic pipe conveying inviscid fluid and lying on an elastic foundation of Winkler type is governed by the partial differential equation (see, e.g., [1-3,7])

$$EI\left(\frac{\partial^4 u}{\partial z^4}+\lambda\frac{\partial^5 u}{\partial z^4 \partial\tau}\right)+MU^2\frac{\partial^2 u}{\partial z^2}+2MU\frac{\partial^2 u}{\partial z\partial\tau}+(m+M)\frac{\partial^2 u}{\partial\tau^2}+c(z)u=0, \tag{1}$$

where $u(z,\tau)$ denotes the transverse displacement of the pipe axis, z – the coordinate along this axis, τ – the time, E – Young's modulus of the pipe material, I – the inertia moment of the pipe cross-section, λ – the internal damping coefficient related to the viscosity of the pipe material, m and M – the masses per unit length of the pipe and the fluid, respectively, U – the flow velocity, and $c(z)$ – the variable foundation modulus.
Let L be the pipe length. Upon introducing the dimensionless parameters

$$x=\frac{z}{L},\quad t=\frac{\tau}{L^2}\sqrt{\frac{EI}{m+M}},\quad w=\frac{u}{L},\quad v=UL\sqrt{\frac{M}{EI}},\quad \beta=\frac{M}{m+M},\quad k(x)=\frac{L^4}{EI}c\left(\frac{z}{L}\right),\quad \eta=\frac{\lambda}{L^2}\sqrt{\frac{EI}{m+M}},$$

where L is the pipe length, Eq. (1) takes the form

$$\frac{\partial^4 w}{\partial x^4}+\eta\frac{\partial^4 w}{\partial x^4\partial t}+v^2\frac{\partial^2 w}{\partial x^2}+2v\sqrt{\beta}\frac{\partial^2 w}{\partial x\partial t}+\frac{\partial^2 w}{\partial t^2}+k(x)w=0. \tag{2}$$

Let the pipe be of cantilevered type, i.e., its end $x=0$ is fixed while the other one, $x=1$, is free. Then, the boundary conditions read

$$w\big|_{x=0}=0,\quad \left.\frac{\partial w}{\partial x}\right|_{x=0}=0,\quad \left.\left(1+\eta\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial x^2}\right|_{x=1}=0,\quad \left.\left(1+\eta\frac{\partial}{\partial t}\right)\frac{\partial^3 w}{\partial x^3}\right|_{x=1}=0 \tag{3}$$

The dynamic behaviour of a cantilevered fluid-conveying pipe is determined by the solutions of the boundary value problem (2), (3).
Here, similarly to [9], we look for the solutions of the BVP (2), (3) of the form

$$w(x,t)=y(x)\exp(\omega t).$$

Substituting this expression into Eq. (2) and boundary conditions (3) one obtains the two-point boundary value problem

$$(1+\eta\omega)\frac{d^4 y}{dx^4}+v^2\frac{d^2 y}{dx^2}+2v\sqrt{\beta}\omega\frac{dy}{dx}+\omega^2 y+k(x)y=0, \tag{4}$$

$$y\big|_{x=0}=0,\quad \left.\frac{dy}{dx}\right|_{x=0}=0,\quad \left.\frac{d^2 y}{dx^2}\right|_{x=1}=0,\quad \left.\frac{d^3 y}{dx^3}\right|_{x=1}=0. \tag{5}$$

Approximate solutions to this problem are obtained using an appropriate modification of the methods in [9] in order to account for the internal damping coefficient.

3. Integral equation
Let us denote by $G(x,\xi)$ the Green function satisfying the differential equation

$$\frac{\partial^4 G}{\partial x^4}=\delta(x-\xi),$$

and the boundary conditions (5) whose jump of the third derivative at $x=\xi$ is 1. It is a simple matter to find that this function is

$$G(x,\xi)=\frac{1}{12}(x^3-3x^2\xi-3x\xi^2+\xi^3)+\begin{cases}\frac{1}{12}(x-\xi)^3, & \text{if} \quad x<\xi;\\ \frac{1}{12}(\xi-x)^3, & \text{if} \quad x\geq\xi.\end{cases}, \tag{6}$$

Using the Green function (6), the solution to the two-point boundary value problem (4)-(5) reads

$$y(x) = \frac{1}{1+\eta\omega}\int_0^1 G(x,\xi)\left\{v^2 \frac{d^2}{d\xi^2} y(\xi) + 2v\sqrt{\beta}\omega\frac{d}{d\xi} y(\xi) + (\omega^2 + k(\xi))y(\xi)\right\}d\xi.$$

Integrating this expression twice by parts it is written as the classic Volterra equation of the second kind

$$y(x) = \int_0^1 \Lambda(x,\xi,\omega,v,\beta)y(\xi)d\xi + w_2 x^2 + w_3 x^3, \tag{7}$$

where the kernel is

$$\Lambda(x,\xi,\omega,v,\beta) = \frac{1}{1+\eta\omega}\left\{v^2(\xi - x) - v\sqrt{\beta}\omega(\xi - x)^2 + \frac{1}{6}(\omega^2 + k(\xi))(\xi - x)^3\right\},$$

and the constants w_2 and w_3 are expressed in terms of the solution $y(x)$ of form

$$w_2 = \frac{1}{1+\eta\omega}\left\{\int_0^1 (v\sqrt{\beta}\omega - \frac{1}{2}(\omega^2 + k(\xi))\xi)y(\xi)d\xi + \frac{1}{2}v\left[(v - 2\sqrt{\beta}\omega)y(1) - v\frac{d}{d\xi}y(\xi)\bigg|_{\xi=1}\right]\right\}, \tag{9}$$

$$w_3 = \frac{1}{1+\eta\omega}\frac{1}{6}\left\{\int_0^1 (\omega^2 + k(\xi))y(\xi)d\xi + v\left[2\sqrt{\beta}\omega y(1) + v\frac{d}{d\xi}y(\xi)\bigg|_{\xi=1}\right]\right\}. \tag{10}$$

Thus, the integral equation (7) with the kernel (8) and expressions (9)-(10) for w_2 and w_3 is equivalent to the two-point boundary-value problem (4)-(5).

The dependence of w_2 and w_3 on $y(x)$ is a feature of (7) that distinguished it from the classic Volterra equation of the second kind. In the present study, approximate solutions to (7) are obtained in the following manner. First, w_2 and w_3 are regarded as given constants independent on $y(x)$ and approximate solutions to the integral equation (7) are obtained by the Neumann series method. These solutions are linear functions of w_2 and w_3. Then, substituting these solutions in expressions (9)-(10) one gets to a system of form

$$a_{11}w_2 + a_{12}w_3 = 0,$$
$$a_{21}w_2 + a_{22}w_3 = 0,$$

admitting a nontrivial solution (w_2,w_3) if and only if

$$\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = 0. \tag{11}$$

The expressions for a_{ij} depend on the frequency ω and all pipe parameters. Thus, given the values of the pipe parameters, the only unknown in (11) is the frequency ω. Solving (11) for ω one obtains the frequencies of the pipe under consideration and can deduce its stability.

Finally, we would like to underline that the Neumann series method is point-wise convergent to the exact solution. Using this property of the Neumann series method, the results obtained by the Galerkin computational procedure are verified determining the corresponding approximate solutions to the integral equation (7). It turned out that the results by the 10-term Galerkin approximation provide an excellent agreement with the results by the integral equation (7).

4. Numerical Results

First, in order to test the aforementioned computational procedure, the critical flow velocities of several well-known problems concerning dynamic stability of cantilevered pipes without foundation have been determined. The results of our computations, shown in Fig. 1, are in an excellent agreement with the earlier results presented in [1] (Fig. 3.30) and [3] (Fig. 8) up to the limiting case $\beta \to 0$ discussed in [9]. Let us recall that elastic pipes are considered in [9]. For such pipes the critical flow velocity v_{cr} depends only on the mass ratio β that is $v_{cr} = v_{cr}(\beta)$. In this case it is established in [9] that the function $v_{cr}(\beta)$ is discontinuous at $\beta = 0$ and the jump is 0.29. In the case under consideration here, the critical flow velocity v_{cr} depends on the internal damping η as well, i.e., $v_{cr} = v_{cr}(\beta,\eta)$. From theoretical point of view, it seems natural to study also the continuity of this function at $\beta = 0$, $\eta = 0$. Similarly to the finding in [9], the function $v_{cr}(\beta,\eta)$ turns out to be discontinuous at this point. Indeed, at $\beta = 0$, $\eta \to 0$, $\eta > 0$ the critical flow velocity tends to $v_{cr} = 3.30$, whereas at $\beta = 0$, $\eta = 0$ it is $v_{cr} = 4.48$. This jump

$$v_{cr}(0,\eta) - \lim_{(\eta\to 0,\eta>0)} v_{cr}(0,\eta) = 4.48 - 3.30 = 1.18$$

is even bigger than the jump in [9].

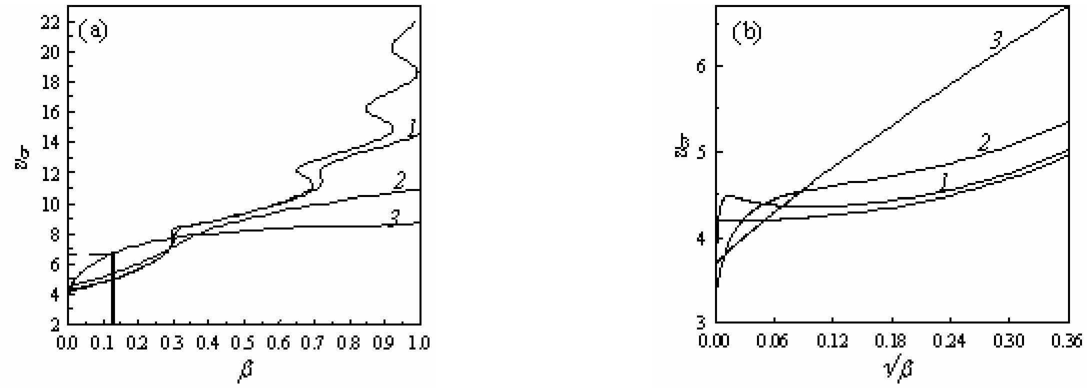

Fig. 1. Critical flow velocity v_{cr} of a cantilevered pipe without foundation $k = 0$ as a function of the mass ratio β at the following four values of the internal damping coefficient η: (a) $\eta = 0$ (thick curve), $\eta = 0.001$ (curve 1), $\eta = 0.01$ (curve 2), $\eta = 0.1$ (curve 3); (b) magnification of the domain marked by the dashed rectangle in figure (a).

In order to clarify the behaviour of the function $v_{cr}(\beta,\eta)$ in a close neighbourhood of the point $\beta = 0$, $\eta = 0$ several cases of very small values of the parameters β and η have been examined. The results are shown in Fig. 2 (a). Observing this figure, one sees that the function $v_{cr}(\beta,\eta)$ behaves quite strange -- for very small values of β and η it varies in the relatively large interval between 3.30 and 4.47. On the other hand, for values of $\sqrt{\beta}$ greater than 0.001, the dependence of the critical flow velocities on the internal damping η, shown in Fig. 2 (b), is not so unusual.

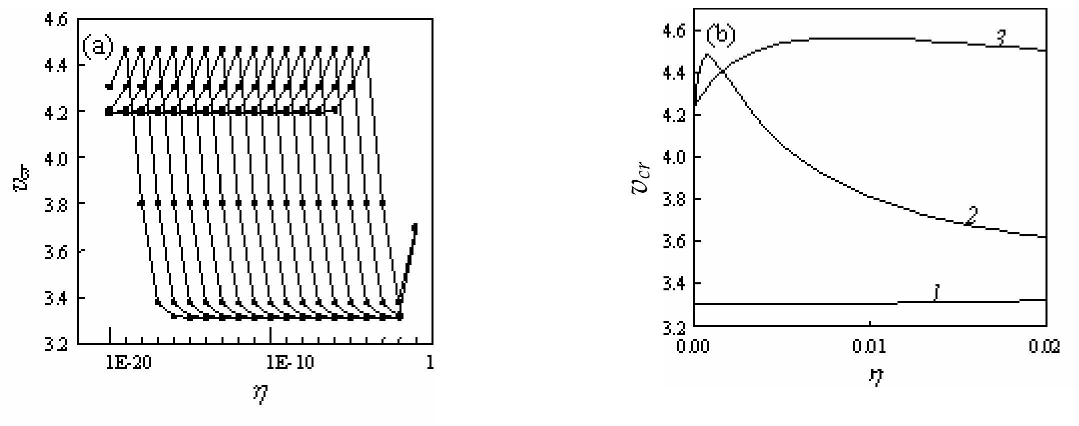

Fig. 2. (a) Values of the critical flow velocity v_{cr} of a cantilevered pipe without foundation $k = 0$ at values η = 1E-20, 1E-19, ..., 1E-1 of the internal damping coefficient at mass ratios $\sqrt{\beta}$ = 1E-18, 1E-17, ..., 1E-3 (from the very left broken line to the very right one, respectively); (b) critical flow velocity v_{cr} of a cantilevered pipe without foundation $k = 0$ as a function of the internal damping coefficient η at mass ratios $\sqrt{\beta} = 0$ (curve 1), $\sqrt{\beta} = 0.01$ (curve 2), $\sqrt{\beta} = 0.1$ (curve 3).

Consider now elastic foundations studied in [9]. Their modulus is a second-order polynomial of the form

$k(x) = 4hx(1 - x)$, $h = const$, $h > 0$,

i.e., it is a concave function with a maximal value h at the middle of the pipe span vanishing at the pipe ends.
These foundations differ from those considered earlier in [7]. The purpose is to study the influence of such foundations on the dynamic stability of viscoelastic pipes.

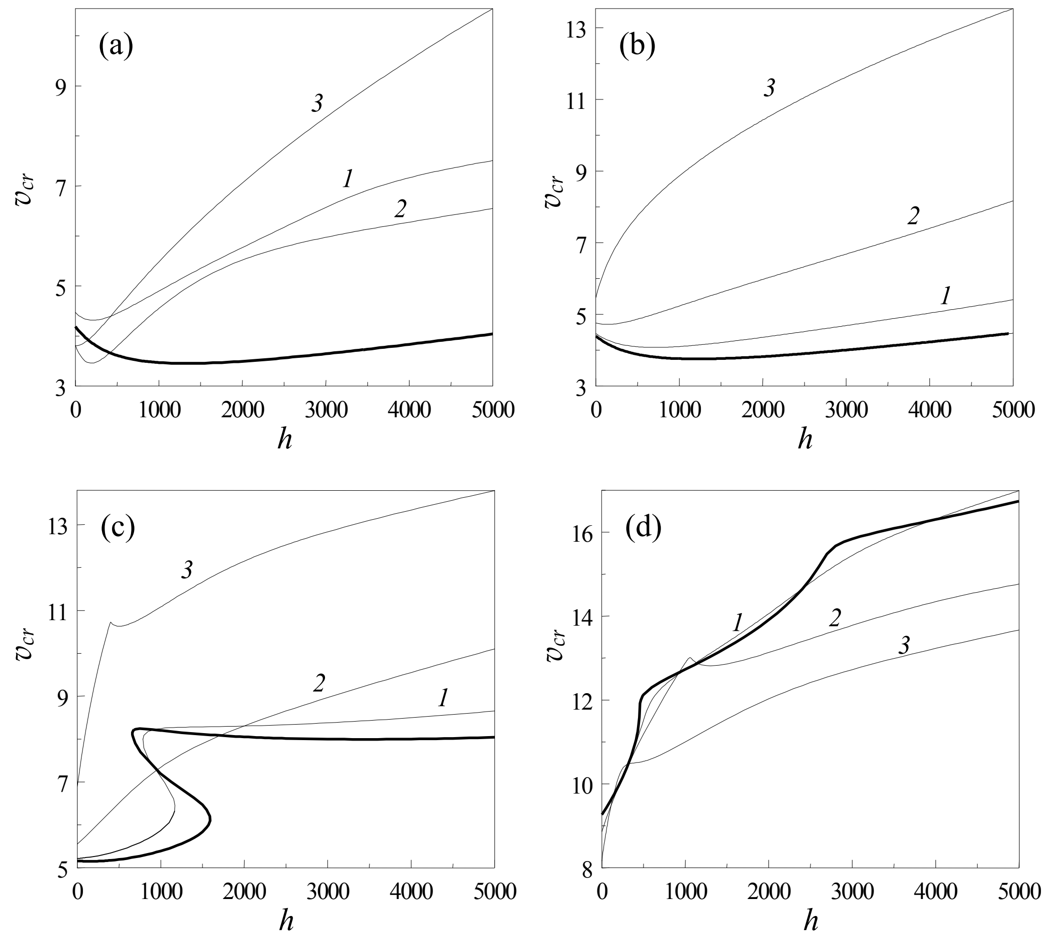

Fig. 3. Critical flow velocity v_{cr} of a cantilevered pipe as a function of the foundation parameter h at values $\eta = 0$ (thick curve), $\eta = 0.001$ (curve 1), $\eta = 0.01$ (curve 2), $\eta = 0.1$ (curve 3) of the internal damping coefficient at mass ratios: (a) $\beta = 0.0001$, (b) $\beta = 0.04$, (c) $\beta = 0.1296$, (d) $\beta = 0.49$.

First, in order to study this influence for small β, the cases $\beta = 0.0001$ and $\beta = 0.04$ are considered. The results for four different values of the internal damping coefficient are shown in Figs. 3 (a) and 3 (b), respectively. Apparently, in the case $\eta = 0.1$ each foundation stabilizes the pipe, but in the other cases a foundation of small h destabilizes the pipe whereas foundations of larger h are stabilizing ones.
Next, pipes of comparatively large mass ratio are considered. The results for pipes with $\beta = 0.1296$ are displayed in Fig. 3 (c). It is seen that at the largest value of the internal damping coefficient $\eta = 0.1$ considered here all foundations have a strong stabilizing effect except for the crease in the vicinity of $h = 500$. Stabilizing effect is observed for $\eta = 0.01$ as well. It should be noted also that the curve corresponding to $\eta = 0.001$ contains an S-shaped domain in the interval $790 \le h \le 1170$, similar to the S-shaped domain in the case of elastic pipe ($\eta = 0$) obtained in [9].

Finally, the critical flow velocities for a pipe with $\beta = 0.49$ are displayed in Fig. 3 (d). It is seen that in the cases $\eta = 0.001$ and $\eta = 0.1$ all foundations considered have a strong stabilizing effect. As for the case $\eta = 0.01$, only foundations such that $1060 < h < 1800$ destabilize the pipe in the sense that the critical flow velocities for such values of the foundation parameter h are less than the critical flow velocity for $h = 1060$.

References

1. Paidoussis, M.: Fluid-Structure Interactions: Slender Structures and Axial Flow, Vol 1. Academic Press, London, 1998.

2. Becker, M., Haugher, W., Winzen, W.: Exact stability analysis of uniform cantilevered pipes conveying fluid or gas, Archives of Mechanics 30 (1978) 757—768.

3(2). Lottati, I, Kornecki, A.: The effect of an elastic foundation and of dissipative forces on the stability of fluid-conveying pipes. Journal of Sound and Vibration 109 (1986) 327-338.

4(3). Doared, O., de Langre, E.: Local and global instability of fluid-conveying pipes on elastic foundations, Journal of Fluids and Structures 16 (1) (2002) 1-14.

5(4). Elishakoff, I., Impolonia, N. :Does a partial elastic foundation increase the flutter velocity of a pipe conveying fluid? Journal of Applied Mechanics - Transactions of the ASME 68 (2001) 206-212.

6(5). Djondjorov, P.: Dynamic stability of pipes partly resting on Winkler foundation, Theoretical and Applied Mechanics 31 (3) (2001) 101-112.

7(6). Djondjorov, P., Vassilev, V., Dzhupanov, V.: Dynamic stability of fluid conveying cantilevered pipes on elastic foundations, Journal of Sound and Vibration 247 (3) (2001) 537-546.

8(7). Djondjorov, P.: On the critical velocities of pipes on variable elastic foundation, Theoretical and Applied Mechanics 31 (4) (2001) 73-81.

9. Vassilev, V., Djondjorov, P.: On the dynamics and stability of fluid-conveying pipes on elastic foundations of variable modulus, Proc. X National Congress on , Theoretical and Applied Mechanics, Varna (2005).

10(8). Mukhin, O. N.: Stability of a pipeline and some methods in non-conservative problems, Vestnik Moskovskogo Universiteta, Series I (Mathematics) 2 (1965), 76-87.