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ON THE DYNAMICS AND STABILITY OF FLUID–CONVEYING ELASTIC PIPES ON ELASTIC FOUNDATIONS OF VARIABLE MODULUS

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1. Introduction

Fluid-conveying pipes are widely used in many industrial branches (nuclear power plants, chemical industry; ship, aircraft and space structures, long-distance pipelines, etc.). Sometimes, their role is simply to transport fluids. In other cases, they function as basic structural components as well. In both cases, however, the dynamic stability of the pipes is crucial for the proper operation of the entire equipment.

Being acknowledged to be of such a significant importance, the dynamic stability of fluid-conveying pipes has been extensively studied in the past 40 years (see, e.g., the comprehensive book by Païdoussis [1] and the references therein). In general, it has been established that if an initially straight pipe conveys inviscid fluid with a relatively low velocity, then each disturbance applied to that pipe causes vibration diminishing with the time. In this case, the initial equilibrium state of the pipe is referred to as a stable one. However, for fluid velocities higher than a certain value (called critical flow velocity) even small disturbances could result in non-diminishing vibration or vibration with larger and larger amplitudes. Under these circumstances, the pipe equilibrium state is referred to as an unstable one.

The dynamic stability of cantilevered pipes on Winkler foundations of constant modulus have been studied by Becker, Haugher and Winzen [2], Lottati and Kornecki [3] and Doare and de Langre [4]. In these works, it has been established that Winkler foundations of constant modulus have a stabilizing effect. Elishakoff and Impolonia [5] and Djondjorov [6] have studied the dynamic stability of cantilevered pipes on Winkler foundations of constant modulus that support only a part of the pipe span. They have obtained that such a foundation can either destabilize or stabilize the pipe depending on the position and length of the foundation. Djondjorov, Vassilev and Dzhupanov [7] and Djondjorov [8] have examined cantilevered pipes on Winkler foundations whose modulus is not constant. These authors have concluded that all such foundations stabilize the pipe.

The present note is concerned with the dynamics and stability of cantilevered fluid-conveying elastic pipes lying on elastic foundations of Winkler type with variable modulus, which differ from those considered in [7] and [8]. The aim is to analyse the effect of the magnitude of the foundation modulus on the dynamic stability of the pipe. For that purpose, a computational procedure based on the Galerkin method is developed for determination of the eigenfrequencies of the pipes and the critical flow velocities. The results obtained are then verified by the shooting method used to solve the respective two-point boundary value problems and by a special technique based on reformulation of the problem in terms of Volterra integral equations.

2. Basic Problem

The small transverse vibration of an initially straight elastic pipe conveying inviscid fluid and lying on an elastic foundation of Winkler type is governed by the partial differential equation (see, e.g., [1-3, 7])

$$EI\frac{\partial^4 u}{\partial z^4} + MU^2\frac{\partial^2 u}{\partial z^2} + 2MU\frac{\partial^2 u}{\partial z \partial \tau} + (m+M)\frac{\partial^2 u}{\partial \tau^2} + c(z)u = 0, \qquad (1)$$

where $u(z,\tau)$ denotes the transverse displacement of the pipe axis, z – the coordinate along this axis, τ – the time, E – Young's modulus of the pipe material, I – the inertia moment of the pipe cross-section, m and M – the masses per unit length of the pipe and the fluid, respectively, U – the flow velocity, and $c(z)$ – the variable foundation modulus.

Upon introducing the dimensionless parameters

$$x=\frac{z}{L},\quad t=\frac{\tau}{L^2}\sqrt{\frac{EI}{m+M}},\quad w=\frac{u}{L},\quad v=UL\sqrt{\frac{M}{EI}},\quad \beta=\frac{M}{m+M},\quad k(x)=\frac{L^4}{EI}c\left(\frac{z}{L}\right),$$

where L is the pipe length, Eq. (1) takes the form

$$\frac{\partial^4 w}{\partial x^4}+v^2\frac{\partial^2 w}{\partial x^2}+2v\sqrt{\beta}\frac{\partial^2 w}{\partial x \partial t}+\frac{\partial^2 w}{\partial t^2}+k(x)w=0. \tag{2}$$

For a pipe of cantilevered type, i.e., its end $x = 0$ is fixed whereas the other one, $x = 1$, is free, the boundary conditions read

$$w\big|_{x=0}=0,\quad \left.\frac{\partial w}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=1}=0,\quad \left.\frac{\partial^3 w}{\partial x^3}\right|_{x=1}=0\,. \tag{3}$$

Thus, the dynamic behaviour of an initially straight elastic pipe conveying inviscid fluid with constant velocity and lying on an elastic foundation of Winkler type of variable modulus is determined by the solutions of the boundary value problem (2), (3).

Here, we look for the solutions of the boundary value problem (2), (3) of the form

$w(x,t)=y(x)\exp(\omega t)$.

Substituting this expression into Eq. (2) and boundary conditions (3) one obtains the following two-point boundary value problem

$$\frac{d^4 y}{dx^4}+v^2\frac{d^2 y}{dx^2}+2v\sqrt{\beta}\omega\frac{dy}{dx}+\omega^2 y+k(x)y=0, \tag{4}$$

$$y\big|_{x=0}=0,\quad \left.\frac{dy}{dx}\right|_{x=0}=0,\quad \left.\frac{d^2 y}{dx^2}\right|_{x=1}=0,\quad \left.\frac{d^3 y}{dx^3}\right|_{x=1}=0. \tag{5}$$

Actually, this constitutes a non-self-adjoint eigenvalue problem, the eigenvalue parameter being the frequency ω.

Here, this eigenvalue problem is solved by a standard Galerkin method, an N-term approximate solution to it being expressed as a linear combination of the first N well-known eigenfunctions of a cantilevered elastic pipe without flow and foundation, i.e., $v = 0$, $k(x) = 0$ (see [1, 7]). The eigenfrequencies are determined as the roots ω_i ($i = 1, 2,..., 2N$) of a $2N$th-order polynomial whose coefficients depend on β, v and some other parameters describing the foundation considered. The critical flow velocities are determined as the lowest values of v at which this polynomial has a root with non-negative real part, the rest of the pipe parameters being kept fixed. Once the values of a critical flow velocity and the corresponding eigenfrequency are obtained for a given number N, a Maple 9.5 implementation of the shooting method (package *shoot*) is applied to check the existence of a sufficiently accurate approximate solution to the respective boundary value problem. The results presented below are achieved using ten terms in the Galerkin approximation of the considered eigenvalue problems. The values of the critical flow velocities and the corresponding eigenfrequencies computed at this level of Galerkin approximation turned out to provide an excellent accuracy of the approximate solutions obtained then by the shooting method, namely: each such solution whose maximal norm is about one satisfies the equation and boundary conditions within an absolute error of order less than 10^{-10}**.**

Setting

$$y(x)=c_0+c_1x+c_2x^2+c_3x^3+\frac{1}{6}\int_0^x(x-\xi)^3\varphi(\xi)d\xi\,, \tag{6}$$

where c_0, c_1, c_2, and c_3 are arbitrary complex numbers, Eq. (4) can be reduced to the Volterra integral equation of second kind

$$\varphi(x)=\int_0^x J(x,\xi)\varphi(\xi)d\xi+f(x), \tag{7}$$

where

$$J(x,\xi)=-(x-\xi)v^2-v\sqrt{\beta}\omega(x-\xi)^2-\frac{1}{6}\left(\omega^2+k(\xi)\right)(x-\xi)^3,$$

$$f(x)=c_0\left(-k(x)-\omega^2\right)+c_1\left(-2\omega v\sqrt{\beta}-\left(k(x)+\omega^2\right)x\right)+$$
$$c_2\left(-2v^2-4\omega v\sqrt{\beta}x-\left(k(x)+\omega^2\right)x^2\right)+c_3\left(-6v^2x-6\omega v\sqrt{\beta}x^2-\left(k(x)+\omega^2\right)x^3\right),$$

For each combination of the complex numbers c_0, c_1, c_2, c_3, pipe parameters and frequency ω., using the recursion formula

$$\varphi_n(x)=\int_0^x J(x,\xi)\varphi_{n-1}(\xi)d\xi+f(x),\quad \varphi_0(x)=f(x),\quad (n=1,2,...),$$

one obtains a sequence $\varphi_n(x)(n=1,2,..)$ of approximate solution of integral equation (7) that converges point-wise to the respective exact solution. Then, on the ground of formula (6), one can construct a sequence $y_n(x)(n=1,2,..)$ of approximate solution of differential equation (4) that also converges point-wise to its exact solution. Now, given an approximate solution $y_n(x)$ and taking into account the boundary conditions (5), one is leaded to a certain matrix $A_{ij}(\omega, v, \beta, k)$ that should to be such that

$A_{ij}(\omega, v, \beta, k)c_j = 0 \quad (i, j = 0, 1, 2, 3)$,

in order that boundary conditions (5) to be satisfied. Consequently, for a given set of values of the pipe parameters, the eigenfrequences ω are determined by the condition

$\det(A_{ij}) = 0$.

The numerical results presented below are also verified using a Maple implementation of the above procedure.

4. Numerical Results

First, in order to test the aforementioned computational procedure, the critical flow velocities of several well-known problems concerning dynamic stability of cantilevered pipes without foundation have been determined. The results of our computations, shown in Fig.1, are in an excellent agreement with the earlier results presented in [1] (Fig.3.30) and [3] (Fig.8) up to the limiting case $\beta \to 0$ to be discussed below.

Let us first note that in the vicinity of $\beta = 1$, for $0.919 \le \beta \le 0.994$, the 10-term Galerkin approximation, verified by the shooting method, predicts that the v_{cr} curve contains an new *S*-shaped domain in addition to the ones presented in [1] (Fig.3.30) and [3] (Fig.8). This observation is in accordance with the remark in the Païdoussis' book [1], "As $\beta \to 1$, more and more *S*-shaped jumps are encountered". On the other hand, Mukhin [9] has shown that at $\beta = 1$ the critical flow velocity tends to infinity. Let us recall that the so-called *S*-shaped domains are associated with an instability-restabilization-instability sequence (see [1]).

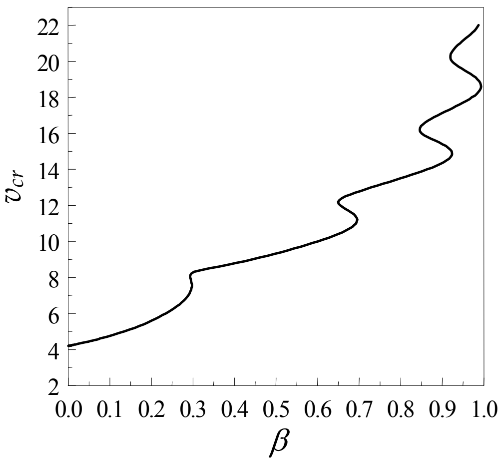

Fig. 1 Critical flow velocity v_{cr} of a cantilevered pipe without foundation ($k = 0$) as a function of the mass ratio β.

As for the vicinity of $\beta = 0$, the results of our computations shown in Fig.1 do not confirm the corresponding curve in [1, 3]. The matter is that for $\beta < 0.01$ this curve in [1, 3] is a straight horizontal line at value $v_{cr} = 4.18$ but in the vicinity of $\beta = 0$ it turns rapidly upward and smoothly goes to $v_{cr} = 4.48$ (see also formulae (16) in [3]). Our computations show that when β approaches zero with positive values the critical flow velocity is $v_{cr} = 4.19$ and the respective curve in Fig.1 never turns upward for β down to 10^{-24}. Therefore, we can conclude that the limit value of the critical flow velocity when $\beta \to 0$, $\beta > 0$ is $v_{cr} = 4.19$, whereas at $\beta = 0$ it is known to be $v_{cr} = 4.48$ (see [1, 3]). For pipes without foundation, the critical flow velocity depends only on the parameter β, that is $v_{cr} = v_{cr}(\beta)$. Thus, it is established that the function $v_{cr}(\beta)$ is discontinuous at $\beta = 0$ and the jump is

$$v_{cr}(0) - \lim_{(\beta \to 0, \beta > 0)} v_{cr}(\beta) = 4.48 - 4.19 = 0.29.$$

This observation contradicts the idea that the critical flow velocity smoothly tends to $v_{cr} = 4.48$ when $\beta \to 0$, $\beta > 0$.

Consider now elastic foundations whose modulus is of the form

$k(x) = 4hx(1 - x)$, $h = const$, $h > 0$, i.e., it is a concave function with a maximal value h at the middle of the pipe span vanishing at the pipe ends. These foundations differ from those considered earlier in [7] and [8]. The purpose is to study the influence of such foundations on the dynamic stability of elastic cantilevered pipes.

First, in order to study this influence for small β, the cases $\beta = 0.0001$ and $\beta = 0.04$ are considered, the results being shown in Fig. 2 (a) and Fig. 2 (b).

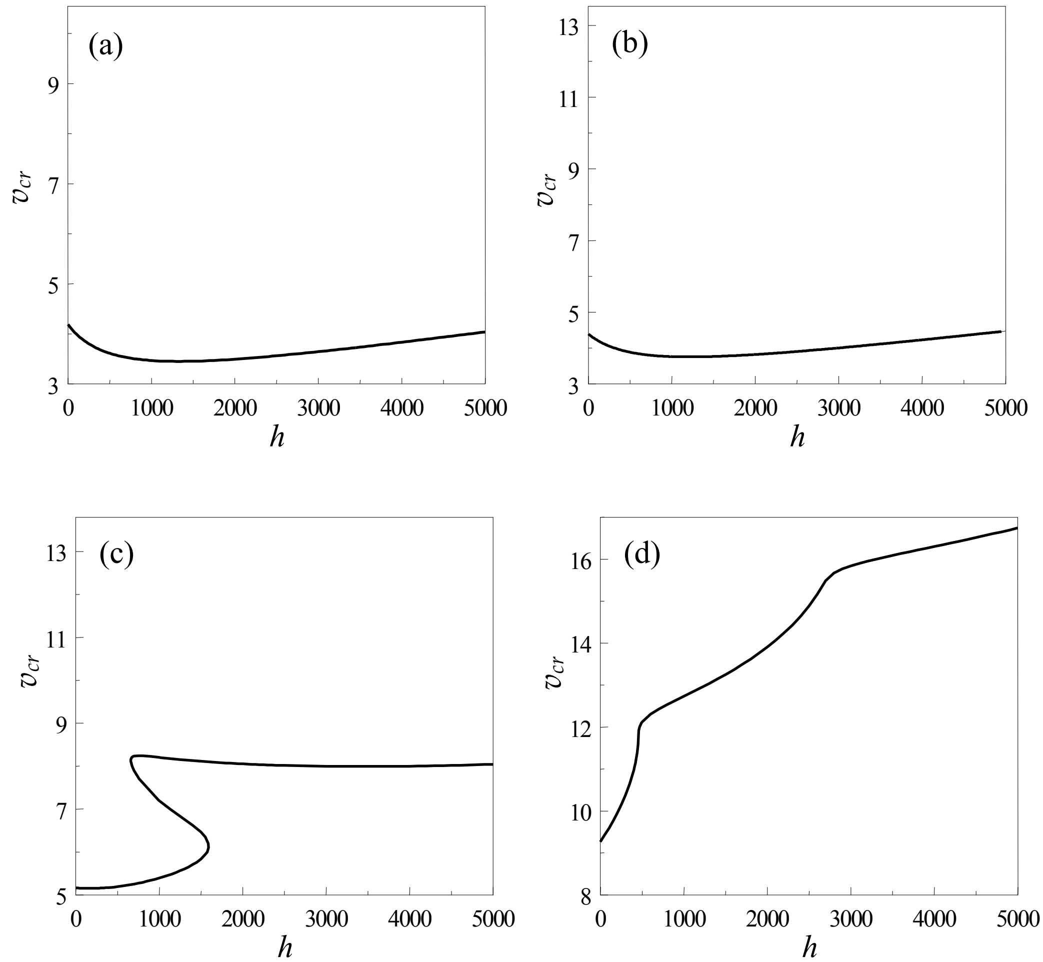

Fig. 2 Critical flow velocity v_{cr} of a cantilevered pipe as a function of the foundation parameter h at the following values of the mass ratios: (a) $\beta = 0.0001$, (b) $\beta = 0.04$, (c) $\beta = 0.1296$, (d) $\beta = 0.49$.

Apparently, foundations of small foundation parameter h destabilize the pipe whereas foundations of larger h are stabilizing ones. For instance, when $\beta = 0.04$, the elastic cantilevered pipe is destabilized for foundations with $h < 4680$, the maximal destabilization effect being achieved at h = 1220 where v_{cr} = 3.75 that is about 85% of the critical flow velocity v_{cr} = 4.39 at h = 0. Thus, due to the influence of an elastic foundation of variable modulus, the critical flow velocity of an elastic cantilevered pipe can be reduced with approximately 15%

Next, pipes of comparatively large mass ratio are considered. The results for a pipe with $\beta = 0.1296$ are displayed in Fig. 2 (c). It is seen that the v_{cr} curve contains an *S*-shaped domain in the interval $680 \le h \le 1600$. Finally, the critical flow velocities for a pipe with $\beta = 0.49$ are displayed in Fig. 2 (d). It is established that all foundations considered have a strong stabilizing effect.

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