INTRODUCTION

There are numerous avenues where legged robots are better to use in comparison to the conventional wheeled ones due their ability to traverse through rough terrains. As a result the immense need of power and complex control laws in embedded systems has also increased to great heights. It is for the same reason that the use of under-actuated robots will help us to traverse through the roughest terrains with minimization of power consumption and application of simple control laws. The present work is a contribution towards the same cause but for the time being it concentrates on rectilinear motion. Most existing four- or eight-legged robots are designed for statically stable operation and stability is assured by keeping the machine's center of mass above the polygon formed by the supporting feet. This, although is the safest mode of locomotion, it brings into picture, the cost of mobility and speed. Furthermore it requires a high mechanical complexity of three degrees of freedom per leg to provide body support during motion.

The quadruped discussed here is supposed to be one of the few like "SCOUT II" [4]. Only few cases of quadruped running robots have been reported in the literature. About 15 years ago, Raibert set the stage with his groundbreaking work on a dynamically stable quadruped, which implemented his three-part controller, via generalizations of the virtual leg idea. The robot featured three hydraulically actuated and one passive prismatic DOF per leg. The robot was able to trot, pace and bound, with smooth transitions between these gaits. Furusho [5] implemented a bounding gait on the 'SCAMPER' robot. Even though the robot's legs were not designed with explicit mechanical compliance, the compliance of the feet, legs, belt transmissions, and the PD joint servo loops were likely significant. The controller divided one complete running cycle into eight states and switched the two joints per leg between free rotation, position control and velocity control. Akiyama and Kimura [6] implemented a bounding gait in the 'PATRUSH' robot. Each three DOF leg featured an actuated hip and knee, and a passive, compliant foot joint.

STATIC FORCE ANALYSIS

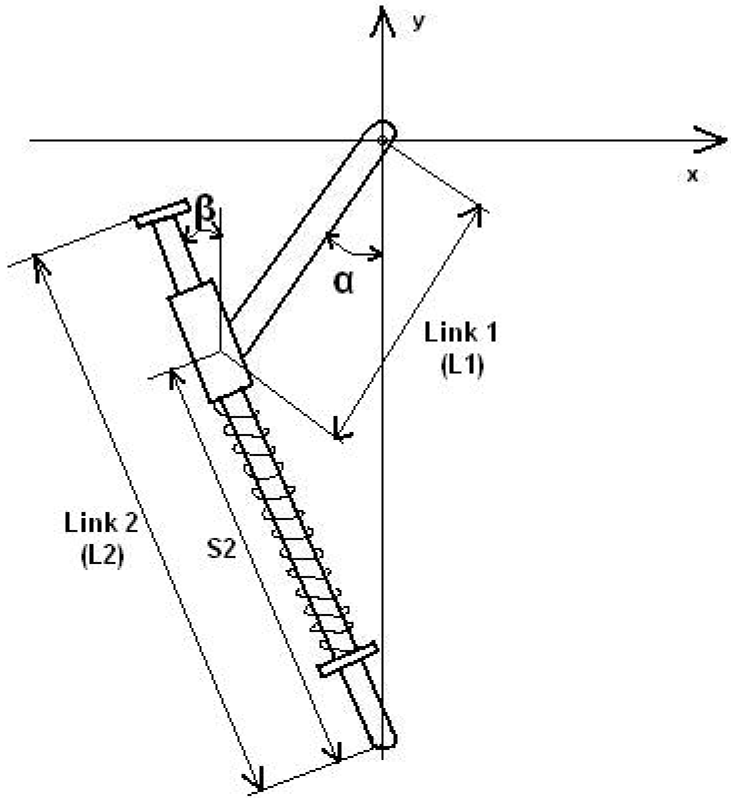


Figure 1

For static translational equilibrium of Link 1:

$W = N' \sin \beta + kx \cos \beta$
$kx \sin \beta = N' \cos \beta$

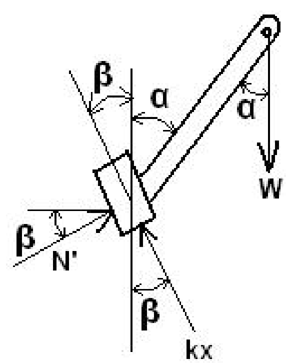


Figure 2

For static translational equilibrium of Link 2:

$$kx = N\cos\beta + f\sin\beta$$
$$N' = N\sin\beta - f\cos\beta$$

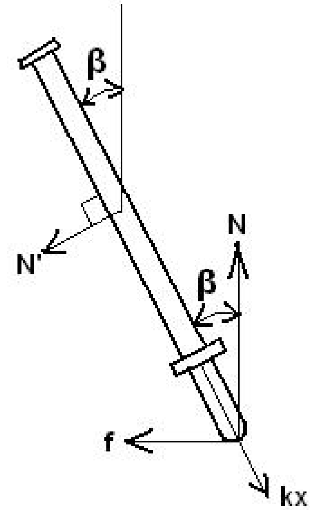


Figure 3

Known Quantities: - k, α, β, W.

Unknown Quantities: - N, N', f, x.

By solving the four equations written above; the four mentioned unknown quantities can be found out.

<u>NOTE</u>: *The Static Force Analysis has been performed for the leg at its initial position.*

KINEMATIC ANALYSIS

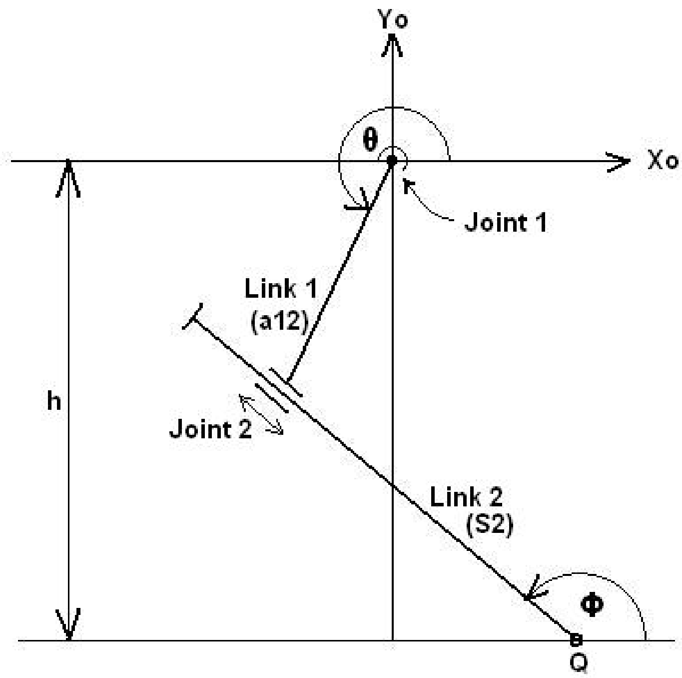


Figure 4

The mechanism shown above is a "Revolute-Prismatic" mechanism and what is going to follow is the development of the link parameters (in accordance with DH Parameter convention) and the Displacement and Velocity analyses.

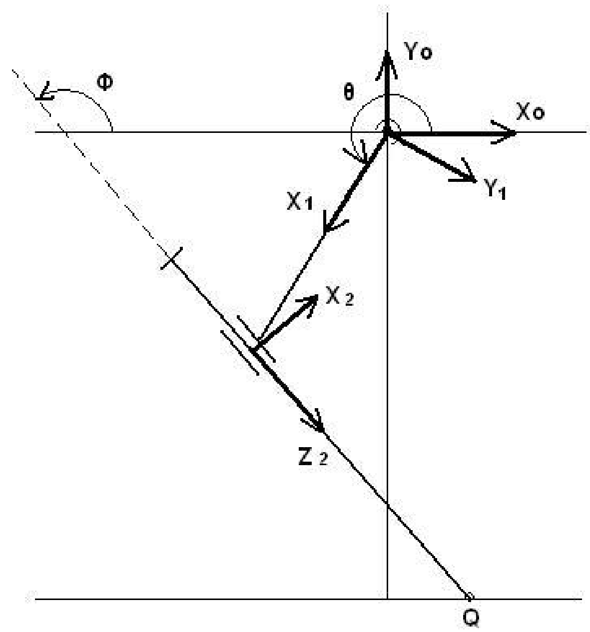


Figure 5

NOTE: *In Figure 5 the axes, Zo and Z_1 are coincident and are in the direction such they come out of the plane of paper and the axis Y_2 is also along the same direction, though originating from different point.*

Link Parameters:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0º	0	0	θ
2	90º	L1	S2	0

Displacement Analysis:

$\mathbf{x_Q = a12\ c\theta + S2\ c\Phi}$
$\mathbf{y_Q = a12\ s\theta + S2\ s\Phi}$

Where: 'c' stands for cosine
's' stands for sine

Variables:

θ (Independent Input Variable)
S2 (Dependant Intermediate/Constrained Variable)
Φ (Dependant Intermediate /Constrained Variable)
x_Q (Output Variable)

Constants:

$a_{12} = L1$
$y_Q = -h$

It is to be noted that by changing the values of 'θ', we will obtain several values of 'x_Q' and that the mechanism is designed such that 'y_Q' remains constant.

Velocity Analysis:

$\mathbf{d(x_Q)/dt = -\Omega\ (a12\ s\theta + S2\ s\Phi) + V\ c\Phi}$
$\mathbf{d(y_Q)/dt = a12\ \Omega\ c\theta + V\ s\Phi + S2\ \Omega\ c\Phi = 0}$ **(since y_Q is constant)**

Where:

- $\Omega = d\theta/dt = d\Phi/dt$ (since $\Phi = \theta - 110^{o}$)
- $V = d(S2)/dt$

Link Transformations:

$$^{0}T_1 = \begin{bmatrix} c\theta & s\theta & 0 & 0 \\ -s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} s(\Phi-\theta) & 0 & -c(\Phi-\theta) & L1 \\ c(\Phi-\theta) & 0 & -s(\Phi-\theta) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Kinematical Simulation of the Leg:

$\theta(^o)$	S2 (cm.)	$\Phi(^o)$	x_Q (cm.)
240	-5.80	130	0.00
245	-5.94	135	1.03
250	-6.15	140	2.14
255	-6.55	145	3.42
260	-7.23	150	4.96

The simulation ahs been done with the following as the initial data required:

L1 = 7.5 cm.
L2 = 9.0 cm.
h = 11 cm.

NOTE: The contents of the table above have been acquired **analytically**.

GEOMETRIC ANALYSIS & LEG DESIGN

Basic Dimensions of the Leg:

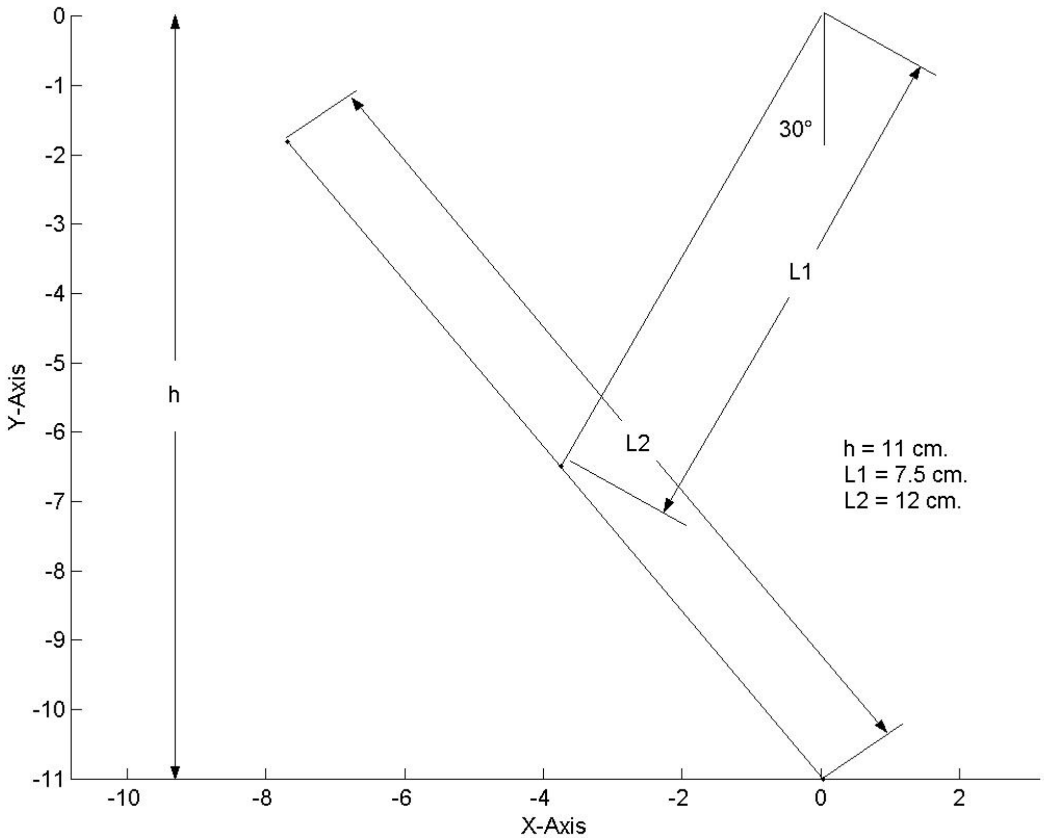


Figure 6

Geometric Simulation:

θ(°)	x_Q(cm.)
240	0.00
245	1.05
250	2.10
255	3.50
260	5.00

Hence for a variation of the input angle θ from 240 ° to 260 °, the Leg Stroke (R) comes out to be 5.00 cm. The adjacent table is a result of practical **geometrical experiments.**

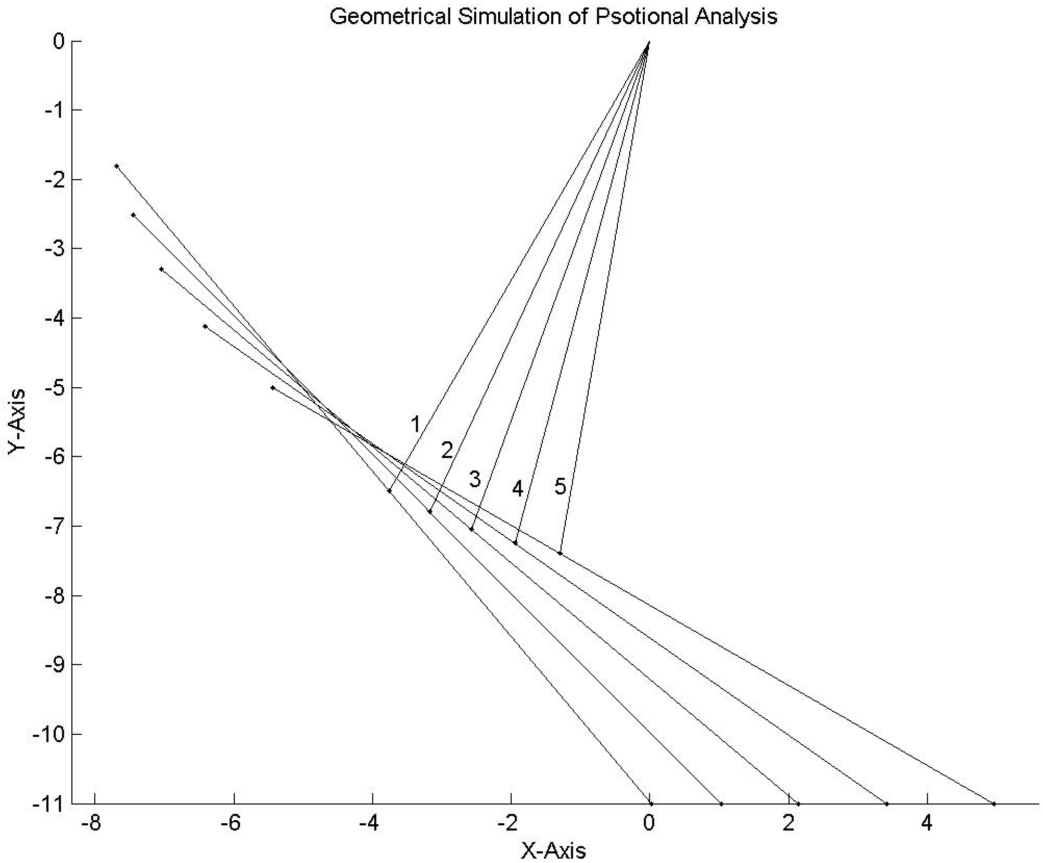


Figure 7

Requirements for Desirable Leg Movement:

1. While moving from position 1 to position 5, the leg must not be in contact with the ground, i.e. it should be in **Transfer Phase**.
2. From position 5 to position 1, the leg should be in contact with the ground and should provide propulsion to the machine.

It directly implies that a mechanism (passive in nature, i.e. without actuation) is required which guides the upper end of link 2 (the end other than that on ground) such that the foot (f) is in the **transfer phase** when link 1 moves from position 1 to position 5 and **support phase** during the return of link 1 from 5 to 1.

Transfer Phase Mechanism Design:

After the commencement of various geometrical experiments with several mechanisms, a mechanism has been selected to solve the purpose of keeping the leg in Transfer Phase during the movement of link 1 from 1 to 5. This has been done keeping in mind, the following factors:

- Commencement of transfer phase keeping link 2 passive.
- Feasibility of the design in geometrical terms.
- Strength of the design.
- Space requirement of the design.
- Minimization of the input torque increment for the actuator.

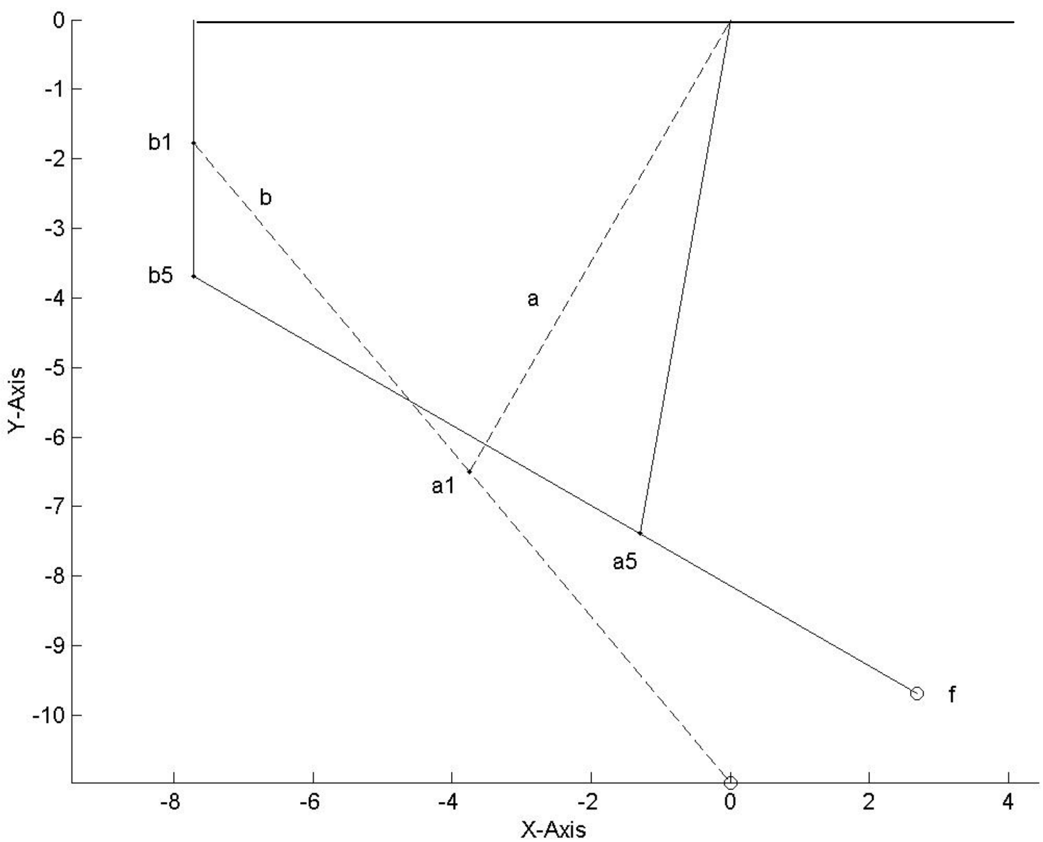
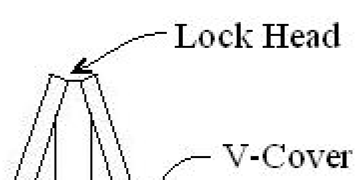


Figure 8

As ‘a’ moves from a1 to a5, ‘b’ moves from b1 to b5 guiding the foot in the Transfer Phase such that ‘f’ is the final position of the foot in accordance with the mechanism used.
The mechanism is basically a solid slotted bar fixed at a distance corresponding to the Support Phase at θ = 240º. The upper end of link 2 is provided with a locking mechanism as shown in the subsequent figures to facilitate the Transfer phase.



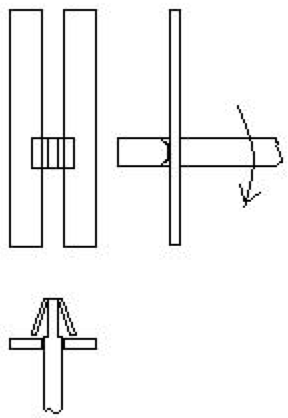


Figure 9

Figure 10

The locking mechanism consists of two thin strips of some plastic material vis-à-vis Perspex fixed at an angle as shown in figure 10. The following figure shows the working of the mechanism.

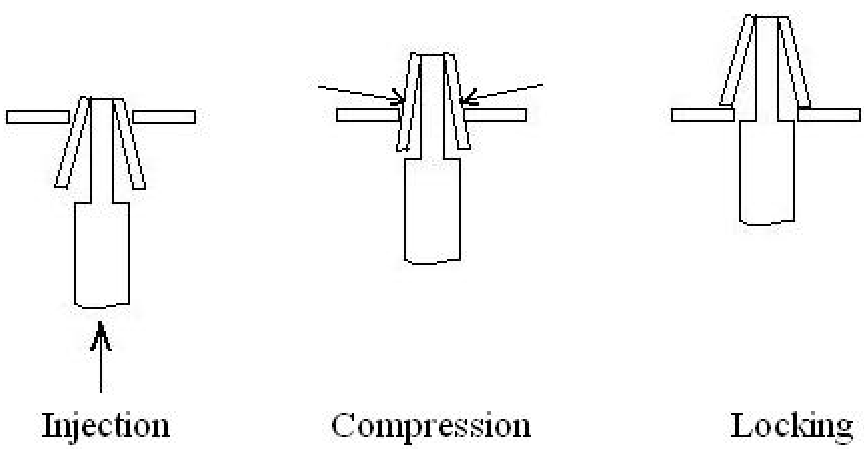


Figure 11

As the Lock Head proceeds towards the slot, the V-Cover gets compressed to an extent such that it is safe from failure. The angle of the inverted 'V' is kept low to an extent that the amount of compression corresponding to the size of the slot through which it has to pass.

This way, the lock makes its way through the slot. After entry, the V-Cover regains its original shape locking the Head beyond the slot.

Sequence of Leg Movements during Gait:

The leg is to be designed so as to follow the motion sequence mentioned in the ensuing lines:

1. All the legs of the quadruped start from θ = 250 ° (i.e. position 3; refer figure 7). The reason behind this arrangement is the gait which the quadruped will use to move from one place to the other.
2. The quadruped is designed to use "Trot Gait". The trot is a steady **2-beat** movement. This gait has a period of suspension. The quadruped springs from one diagonal to the other. In between these springs, all four legs are off the ground. The quadruped is supposed to lift its hooves as follows:

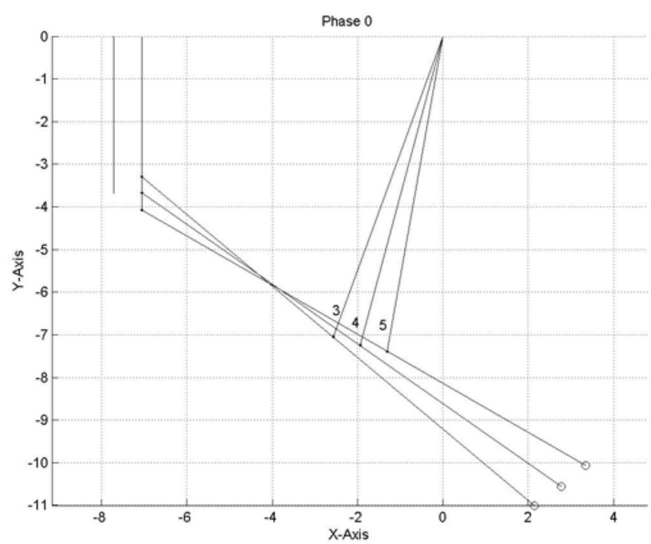
   (1st beat) **right fore /left hind**

   (2nd beat) **left fore / right hind**

   i.e. one set of legs across one of the diagonals goes from position 3 to 5 (refer figure 7) and the other set from position 3 to 1.

3. It is to be noted that from position 3 to 5 also, the leg should be in 'Transfer Phase'. Hence, another slotted bar has to be placed at an appropriate position (it is the distance along –x-axis corresponding to the position of the upper end of the foot at θ = 250 °.

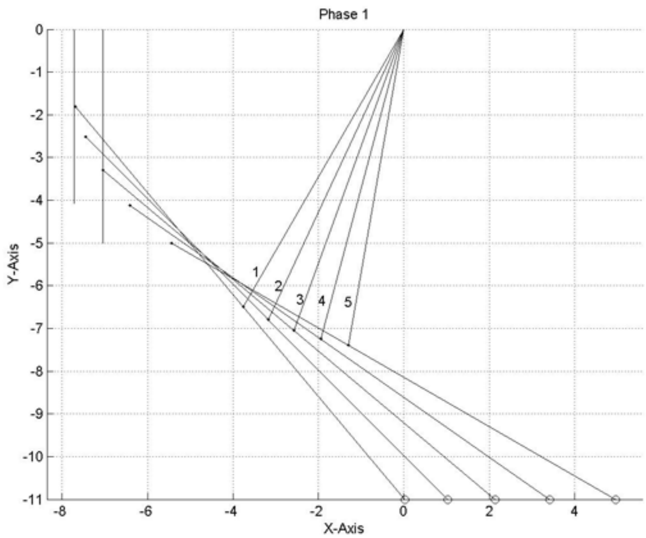
Keeping in mind, the points mentioned above the motion of the leg can be divided into the following phases:



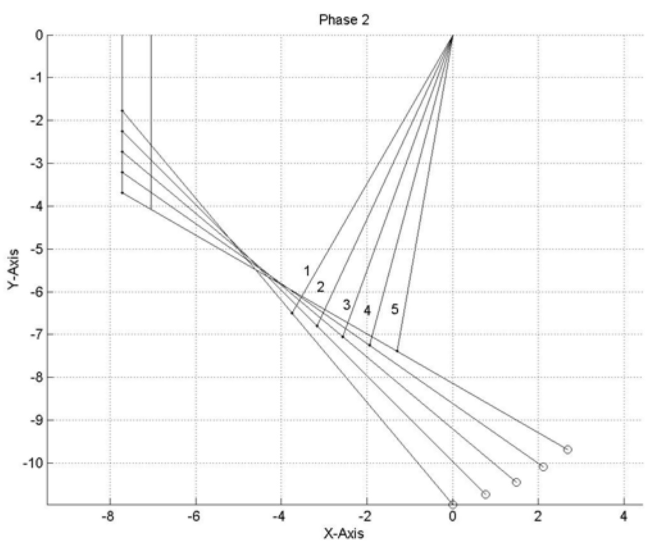
Phase 0

θ(°)	x_Q(cm.)	y_Q(cm.)
250	2.10	-11.0
255	2.75	-10.5
260	3.40	-10.0

θ(°)	x_Q(cm.)	y_Q(cm.)
260	5.00	-11.00
255	3.50	-11.00
250	2.10	-11.00
245	1.05	-11.00
240	0.00	-11.00



Phase 1



Phase 2

θ(°)	x_Q(cm.)	y_Q(cm.)
240	0.00	-11.00
245	1.00	-10.50
250	1.65	-10.45
255	2.35	-10.10
260	2.75	-09.65

During the rest of the gait, the leg sequence will oscillate between Phase 1 and Phase 2.

Final Leg Design:

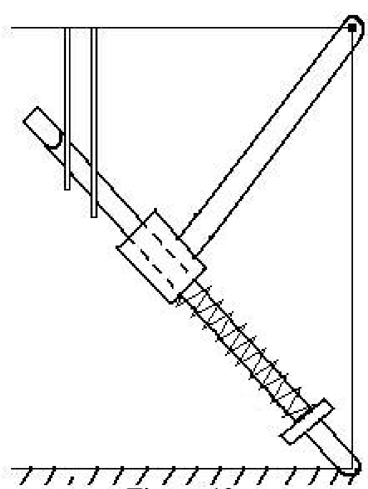


Figure 12

MODIFICATIONS IN KINEMATIC FORMULATIONS

As witnessed earlier, the final leg design is quite different from the basic design due to the inculcation of the mechanical arrangement for the commencement of the Transfer Phase when the input angle (θ) increases in anti-clockwise sense.

Hence there are three stages in which the leg motion takes place which have been discussed earlier under headings Phase 0, Phase 1 and Phase 2.

It is hence desirable to perform separate kinematical analysis for all the three phases. Another point to note here is that all the three phases share the same set of link parameters i.e.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0º	0	0	θ
2	90º	L1	S2	0

Phase 0 (Initial Transfer Phase):

Transition- from position 3 to 5 (refer figure 7)
Displacement Analysis:

$$\mathbf{x_Q = L2[\ c(180°+\Phi\ ) - c(40°)\ ] + x_{Qo}}$$
$$\mathbf{y_Q = L1\ s\theta + (L2 + S2')\ s(180°+\Phi)}$$

Independent Variable(s):

θ Input Angle (varying between 240°-260°)

Dependant Variables:

$\Phi = \theta + 110°$
$S2' = \{x_{Qo} - L2\ c(40°) - L1\ c\ \theta\}/c(180° + \Phi)$
x_Q, y_Q

Constants:

L1 = 7.5 cm.
L2 = 12.0 cm.
x_{Qo} = 2.1450 cm.

Velocity Analysis:

$d(x_Q)/dt = -L2\ \Omega\ s(180° + \Phi)$
$d(y_Q)/dt = L2\ \Omega\ c(180°+ \Phi) + V'\ s(180°+\Phi) + S2'\ \Omega\ c(180°+\Phi) + L1\ \Omega\ c\ \theta$

Where:

- $\Omega = d\theta/dt = d\Phi/dt$ (since $\Phi = \theta-110°$)
- $V' = d(S2')/dt$

Phase 1 (Support Phase):

Transition- from position 5 to 1 (refer figure 7)
Displacement Analysis:

$x_Q = L1\ c\theta + S2\ c\Phi$
$y_Q = L1\ s\theta + S2\ s\Phi$

Independent Variable(s):

θ Input Angle (varying between 240°-260°)

Dependant Variables:

$\Phi = \theta + 110°$
S2
x_Q, y_Q

Constants:

L1 = 7.5 cm.
L2 = 12.0 cm.
x_{Qo} = 2.1450 cm.

Velocity Analysis:

$d(x_Q)/dt = -\Omega\ (L1\ s\theta + S2\ s\Phi) + V\ c\Phi$
$d(y_Q)/dt = L1\ \Omega\ c\theta + V\ c\Phi + S2\ \Omega\ c\Phi = 0$ (since y_Q is constant)

Where:

- $\Omega = d\theta/dt = d\Phi/dt$ (since $\Phi = \theta - 110^{o}$)
- $V = d(S2)/dt$

Phase 2(Transfer Phase):

Transition- from position 1 to 5 (refer figure 7)

Displacement Analysis:

$$\mathbf{x_Q = L2[\ c(180^\circ+\Phi\ ) - c(50^\circ)\ ] + x_{Qo}}$$
$$\mathbf{y_Q = L1\ s\theta + (L2 + S2')\ s(180^\circ+\Phi)}$$

Independent Variable(s):

θ Input Angle (varying between 240°-260°)

Dependant Variables:

$\Phi = \theta + 110^\circ$

$S2' = \{ - L2\ c(50^\circ) - L1\ c\ \theta\}/c(180^\circ + \Phi)$

x_Q, y_Q

Constants:

L1 = 7.5 cm.

L2 = 12.0 cm.

x_{Qo} = 2.1450 cm.

Velocity Analysis:

$$\mathbf{d(x_Q)/dt = -L2\ \Omega\ s(180^\circ + \Phi)}$$
$$\mathbf{d(y_Q)/dt = L2\ \Omega\ c(180^\circ+ \Phi) + V'\ s(180^\circ+\Phi) + S2'\ \Omega\ c(180^\circ+\Phi) + L1\ \Omega\ c\ \theta}$$

Where:

- $\Omega = d\theta/dt = d\Phi/dt$ (since $\Phi = \theta - 110^{o}$)
- $V' = d(S2')/dt$

It can be very well seen here that the equations of phase 0 and phase 2 are similar to a great extent. It is due to the fact that both of them originate from the same array of mathematical idea with a difference in just the location of the phenomenon.

Kinematical Simulation & Comparison with Geometrical Results:

Phase 0

θ(°)	S2'(cm.)	x_Q(cm.)	y_Q(cm.)
250	-5.85	2.145	-11.00
255	-6.23	2.782	-10.55
260	-6.63	3.344	-10.07

Phase 1

θ(°)	S2(cm.)	x_Q(cm.)	y_Q(cm.)
260	-7.23	4.95	-11.00
255	-6.54	3.42	-11.00
250	-6.15	2.14	-11.00
245	-5.94	1.03	-11.00
240	-5.88	0.03	-11.00

Phase 2

θ(°)	S2'(cm.)	x_Q(cm.)	y_Q(cm.)
240	-6.17	0.00	-10.96
245	-6.42	0.77	-10.73
250	-6.72	1.48	-10.44
255	-7.04	2.11	-10.08
260	-7.40	2.67	-09.68

On comparison of the results above with the geometrical results deduced previously, the consistency of the results can be well observed with the maximum error of 2%.

Experimental Setup:

First and foremost it is desirable to test the leg-design, specifically the locking mechanism before going for the whole quadruped. This can be done by making use of "Cart-wheel" arrangement, i.e. replacing the rear legs with two wheels without providing them actuation. The leg components have been cut out of half inch thick perspex sheet and the prismatic joint has been made by quarter inch thick sheet of perspex.
For actuation two 12V D.C. stepper motors have been used with a reduction of 20:1 using two worm-worm wheel pairs. The setup is controlled by a programmed 8051 microcontroller chip.

Gear Arrangement for Quadruped:

The following figure shows the top-view of the prototype of the quadruped. The spur gear at the center is provided with actuation and all the output shafts move simultaneously in accordance with the gait planned.

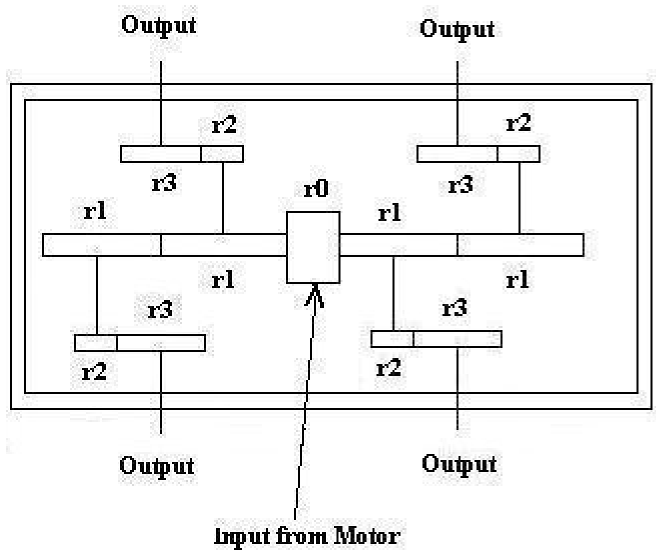


Figure 13

r0, r1, r2 and r3 are the respective radii of the spur gears as shown in figure 13. It can be observed with a little exercise that on rotating r0 in a particular direction, say clockwise, two adjacent outputs rotate in directions opposite to each other and the outputs along any diagonal rotate in the same sense (i.e. zero phase difference), hence creating a motion of link 1 of all the legs appropriate enough to let the quadruped move according to the decided gait.

Discussion:

From all the analysis that has been done till now, the following points need to be kept in mind for a better overview of the system as a whole.

- The leg design still has room for modifications and development as the locking mechanism which has been used to facilitate the transfer phase, is one of the major sources of the friction in the system and also the input torque to the motor.
- During the whole process of designing the leg of the quadruped, all the procedures used aim for one thing and that is the rectilinear motion of the quadruped without the use of more than one actuator.

This implies that the control of the system is easy but the design is bulky and somewhat complicated since it has to compensate for the ease of electronic control.

- One of the most important aspects of the design is the gait of the quadruped. Since ‘trot gait’ is being used, it is to be noted that there will be no ‘support polygon’ for the quadruped as at any instance two legs along one of the diagonals will be in air and for the stability of the system the centre of gravity of the system has to be in the diagonal which represents the legs in support phase. This can be done either by using dynamic methods for stability or by increasing the speed of the transition from transfer phase to support phase. This can be done by increasing the angular velocity Ω of link 1.
- By increase of the velocity the torque will decrease. Hence the actuator used should be strong enough to provide high torque even at high speeds.
- The overall weight of the system should be less so as to reduce the chances of the failure of any of the leg component.

Conclusion:

The present work has mainly concerned itself with the kinematical and geometric analyses of the leg design which is supposed to be the most important portion of the work. The paper also consists of the illustration of the gear-trains designed specifically for providing actuation to all the legs with a single actuator. Although the experimental setup has not yet been completed, the geometrical results are compatible with the analytical results. However, the completion of the experimental setup for both, the leg design as well as the quadruped would provide a better practical observation for comparison.

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