Exploration of a Double-Tapered Disc-Loaded Circular Waveguide for a Wideband Gyro-TWT

Vishal Kesari, P. K. Jain, and B. N. Basu

***Abstract*—The analysis of a disc-loaded circular waveguide interaction structure of a gyro-traveling-wave-tube (gyro-TWT) considering standing and propagating mode harmonics in the disc-occupied and disc-free regions, respectively, gave the beam-absent dispersion relation of the waveguide. The axial phase propagation constant predicted by the dispersion relation was substituted into the gyro-TWT gain-equation, the latter obtainable from the beam-present dispersion relation of the device. A method of double-tapering the structure dimensions was proposed that consists in tapering the disc-hole radius to distribute the midband frequency of amplification over a wide range of frequencies, and simultaneously tapering the waveguide-wall radius to compensate for gain reduction at band edges due to disc-hole radius tapering. The method has demonstrated wide device bandwidths at relatively large gain values.**

***Index Terms*—Disc-loaded waveguide, millimeter-wave amplifier, periodic electromagnetic structure, wideband gyro-traveling-wave-tube (gyro-TWT).**

I. Introduction

SUCCESSFUL development of long-range and high-resolution radar and high information density communication systems in the millimeter-wave frequency led to efforts in finding methods for widening the bandwidth of a gyro-traveling-wave tube (gyro-TWT) [1]–[11]. In one of these methods, in which the waveguide cross section is tapered and at the same time the magnetic field is profiled [2]–[6], although the bandwidth is increased due to different portions of the interaction length of the tapered cross section waveguide becoming effective for different frequency ranges, irrespective of whether the cross section is up-tapered toward the collector or gun end of the device, the gain of the device decreases due to a reduction in the effective interaction length at each of such frequency ranges [2]–[6]. In another method, the dispersion characteristics of the waveguide are controlled for wideband coalescence between the beam-mode and waveguide-mode dispersion characteristics [7]–[11], for instance, by disc loading a circular waveguide and controlling the disc parameters [7]–[9].

The objective of the present letter is to propose a method, which accrues the advantages of both the above methods of broadbanding a gyro-TWT. In the proposed method, the disc parameters are optimized for wideband coalescence bandwidth and at the same time the disc parameters, and more precisely, the disc-hole and waveguide-wall radii are tapered, expecting that the reduction of the device gain due to tapering of structure cross section for wider device bandwidths [2]–[6] would be compensated for by the enhancement of the structure interaction impedance caused by the presence of the discs, more precisely by the decrease of disc-hole radius amounting to an increase of the amount of disc area [11].

II. Analysis

The disc-loaded waveguide was analyzed by several approaches, for instance, by the surface impedance model for the transverse magnetic (TM) mode, for closely spaced discs [12]; the coupled-integral-equation technique for the transverse electric (TE) mode [13] as well for hybrid TE and TM modes [14]; the modal expansion technique for hybrid TE and TM modes [15]; transfer matrix and half-cell formulations [16]; and modal field matching technique [17]. Keeping the potential application of broadbanding a gyro-TWT in mind, Choe and Uhm [7] analyzed a disc-loaded circular waveguide in the fast-wave regime, including the effects of the lowest order standing-wave mode in the disc-occupied region and the fundamental, traveling-wave mode in the disc-free region, however excluding the effects of finite disc thickness. Kesari *et al.* [8], [9] extended the analysis of Choe and Uhm [7] considering higher order modes in the disc-free and disc-occupied regions, and ignoring [8] as well as considering [9] the effects of finite disc thickness.

The structure may be tapered linearly with respect to the four parameters, namely, waveguide-wall radius r_W, disc-to-disc distance L, disc-hole radius r_D, and disc thickness T. The continuous tapering of the length of the structure is approximated by the variation of these parameters over the length l of the structure in n_D number of steps, being equal to the number of discs: $1 \leq p \leq n_D$, a disc being positioned at the middle of a step, where p refers to a step. Here, $p = 1$ and $p = n_D$ refer to the start and end steps of the structure, respectively (Fig. 1). It is also implied that, within a particular step, the four structure parameters r_W, L, r_D, and T are each uniform, defined as (1a), shown at the bottom of the next page.

Following [9], the dispersion relation of the structure referring to the pth step, excited typically in an azimuthally symmetric TE mode, may be written as

$$\det \left| M_{\mathrm{nm}} J_0 \left\{ \gamma_n^I r_{D,p} \right\} Z'_0 \left\{ \gamma_m^{\mathrm{II}} r_{D,p} \right\} - Z_0 \left\{ \gamma_m^{\mathrm{II}} r_{D,p} \right\} J'_0 \left\{ \gamma_n^I r_{D,p} \right\} \right| = 0 \quad (2)$$

where [shown in (1b) at the bottom of the page]the νth solution of which would correspond to the $\mathrm{TE}_{0\nu}$ mode. J_0 and Y_0 are the

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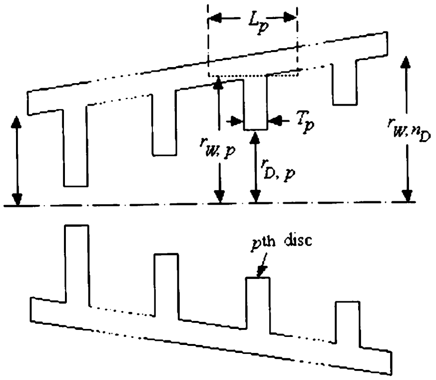

Fig. 1. Longitudinal cross section of a double-tapered disc-loaded circular waveguide that is tapered with respect to the waveguide-wall and disc-hole radii.

zeroth-order Bessel functions of the first and second kinds, respectively. The prime with a function represents the derivative of the function with respect to its argument. $\gamma_n^I(=[k^2-(\beta_n^I)^2]^{1/2})$ and $\gamma_m^{\rm II}(=[k^2-(\beta_m^{\rm II})^2]^{1/2})$ are the radial propagation constants in terms of the axial phase propagation constants β_n^I and $\beta_m^{\rm II}$, respectively, where the superscripts *I* and *II* refer to the disc-free and disc-occupied regions, respectively, k being the free-space propagation constant.

Referring to the pth step of tapering, one may express β_n^I as $\beta_n^I = \beta_0^I + 2\pi n/L_p (n = 0, \pm 1, \pm 2, \ldots, \pm\infty)$, according to Floquet's theorem, in the disc-free, free-space region supporting propagating space-periodic mode. Similarly, one may express $\beta_m^{\rm II}$ as $\beta_m^{\rm II} = m\pi/(L_p - T_p)(m = 1, 2, 3, \ldots, \infty)$ in the freespace region of the groove between discs of axial length $L_p - T_p$ supporting stationary waves of integral multiple of half guide wavelengths [7]–[9], [11].

One may estimate the effect of tapering the structure parameters on the performance of a gyro-TWT with the help of the small-signal gain-equation of the device, which in turn is obtainable from the beam-present dispersion relation of the waveguide [10] in the presence of gyrating electrons [6], [9]. Thus, one may use the following approximate expression for the device gain G in decibels, ignoring launching loss and adding contributions G_p ($p = 1$ to n_D) from all the steps of the step-tapered gyro-TWT [6], [9], [10]

$$G \cong \sum_{p=1}^{p=n_D} G_p = \sum_{p=1}^{p=n_D} B_p C_p N_p \tag{3}$$

where [shown in (1c) at the bottom of the page] where x_1 is the real part of the solution of the cubic equation $\delta(\delta + jb)^2 = j$ having a positive value [6], [9], [10], $b(= (\omega - \beta_0^I v_{z,p} - s\omega_c/\gamma)/(\beta_0^I v_{z,p} C_p))$ being a synchronization parameter, where s is the beam-harmonic mode number, ω_c is the nonrelativistic cyclotron angular frequency, and $\gamma(= 1 + |e|V_0/m_{e0}c^2)$ is the relativistic mass factor, V_0 being the beam voltage, e electronic charge, m_{e0} rest mass of an electron and c the velocity of light in freespace, $v_{z,p}(= [(\gamma^2-1)/(1+\alpha_0^2)]^{1/2}/\gamma)$ is the axial velocity of electrons, and $\alpha_0(= v_{t,p}/v_{z,p})$ the beam pitch factor $v_{t,p}$ being the transverse velocity of electrons. N_p is the interaction length L_p of the pth step measured in terms of guide wavelengths $2\pi/\beta_0^I$. I_0 is the beam current. $r_{H,p}$ and $r_{L,p}$ are the hollow-beam radius and the Larmor radius in the pth step, respectively, [6], [9], [10].

III. Results and Discussion

In general, one has option to choose either one or simultaneously more than one of the tapering parameters $r_{W,p}, L_p, r_{D,p}$, and T_p according to (1). The axial phase propagation constant β_0^I for a given frequency $f(= \omega/2\pi)$, obtainable from (2), may be substituted in (3) for the gain of the gyro-TWT [6], [9], [10]. Synchronously profiled magnetic flux density $B_{0,p}$ of the pth step maintains, throughout the structure length, a constant ratio with the corresponding grazing-point value $B_{g,p}(= m_{e0}\gamma\omega_{{\rm cut},p}/(|e|\, s\gamma_{z,p}))$ in terms of the cutoff angular frequency $\omega_{{\rm cut},p}$ and $\gamma_{z,p} = (1 - v_{z,p}^2/c^2)^{-1/2}$ of the pth step (2), [6], [9]. Further, the beam parameters $r_{L,p}, r_{H,p}$, and $v_{t,p}$

$$\left.\begin{aligned} r_{W,p} &= r_{W,1} + (r_{W,n_D} - r_{W,1})(p-1)/(n_D-1) && \text{(a)}\\ L_p &= L_1 + (L_{n_D} - L_1)(p-1)/(n_D-1) && \text{(b)}\\ r_{D,p} &= r_{D,1} + (r_{D,n_D} - r_{D,1})(p-1)/(n_D-1) && \text{(c)}\\ T_p &= T_1 + (T_{n_D} - T_1)(p-1)/(n_D-1) && \text{(d)} \end{aligned}\right\} \tag{1a}$$

$$\begin{aligned} M_{\rm nm} &= \frac{\gamma_n^I \beta_m^{\rm II}\left[1 - (-1)^m \exp\left(-j\beta_0^I L_p\right)\right]}{\gamma_m^{\rm II}\left[\beta_m^{\rm II} - \exp\left(-j\beta_0^I L_p\right)\left(\beta_m^{\rm II}\cos\left(\beta_m^{\rm II}L_p\right) + j\beta_n^I \sin\left(\beta_m^{\rm II}L_p\right)\right)\right]}\\ Z_0\left\{\gamma_m^{\rm II}r\right\} &= \left(Y_0'\left\{\gamma_m^{\rm II}r_{W,p}\right\}J_0\left\{\gamma_m^{\rm II}r\right\} - J_0'\left\{\gamma_m^{\rm II}r_{W,p}\right\}Y_0\left\{\gamma_m^{\rm II}r\right\}\right)/Y_0'\left\{\gamma_m^{\rm II}r_{W,p}\right\} \end{aligned} \tag{1b}$$

$$\begin{aligned} B_p &= 40\pi(\log_{10} e)x_1 \cong 54.6x_1\\ C_p &= \left[\frac{I_0}{4V_0}\,\frac{(\mu_0/\varepsilon_0)^{1/2}(v_{t,p}/c)^2\left(\gamma_0^I r_{W,p}\right)^2\left(1+\alpha_0^2\right)J_s^2\left\{\gamma_0^I r_{H,p}\right\}J_s'^2\left\{\gamma_0^I r_{L,p}\right\}}{\pi J_0^2\left\{\gamma_0^I r_{W,p}\right\}(v_{z,p}/c)\left(\beta_0^I r_{W,p}\right)^4}\right]^{1/3}\\ N_p &= \beta_0^I L_p/2\pi \end{aligned} \tag{1c}$$

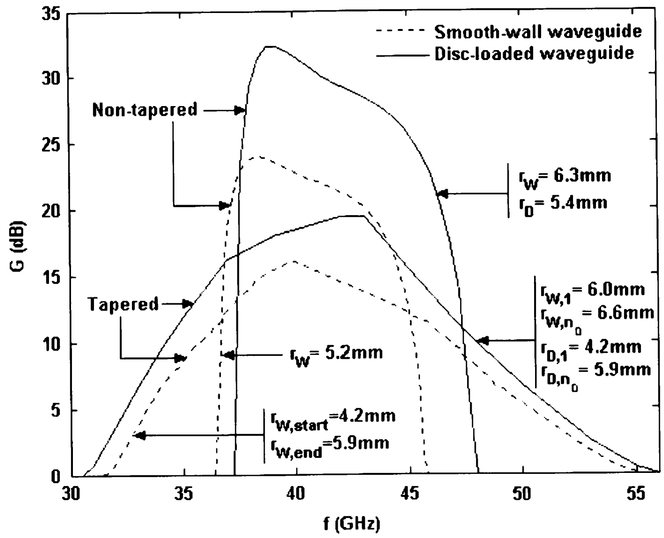

Fig. 2. Gain-frequency plots of a gyro-TWT in tapered and nontapered disc-loaded and smooth-wall circular waveguides, taking typically the excitation mode as TE_{01} and the interaction length as $l = 162$ mm. The typical structure parameters are $n_D = 54, T_p = 1.0$ mm ($p = 1$ to n_D) and $L_p = 3.0$ mm ($p = 1$ to n_D) for the disc-loaded waveguide; and $T = 1.0$ mm and $L = 3.0$ mm, for the smooth-wall waveguide, with the start and end values $r_{W,\mathrm{start}}$ and $r_{W,\mathrm{end}}$, respectively, following the taper profile of the disc-hole radius of the corresponding tapered disc-loaded waveguide. Typical beam and magnetic field parameters for the plots are mentioned in the text.

have to be so taken as to maintain a constancy of each of the parameters $r_{L,p}B_{0,p}^{1/2}, r_{H,p}B_{0,p}^{1/2}$ and $v_{t,p}B_{0,p}^{-1/2}$ over all the steps, in order to obey the adiabatic beam-flow condition [18], the conservation of magnetic flux [5], and the conservation of electron magnetic moment [5], respectively.

One may thus generate the gain-frequency plots of a gyro-TWT in a disc-loaded circular waveguide with the help of (2) and (3), typically for the TE_{01} mode with a chosen scheme of tapering according to (1) (Fig. 2). For this purpose, the beam parameters are taken typically as $I_0 = 9.0\mathrm{A}, V_0 = 100$ kV, and $\alpha_0 = 0.5$. The magnetic field parameters are taken typically such that, for the tapered structure, at the starting of taper: $r_{H,1} = 3.5$ mm, $r_{L,1}/r_{W,1} = 0.1$, and through out the taper: $B_{0,p}/B_{g,p} = 1.0$ ($p = 1$ to n_D), which all continue to be the same for the nontapered structure, interpreted as uniform throughout the interaction length (Fig. 2).

Interpreting G_p to be the same for all the steps ($p = 1$ to n_D) in (3), one may plot the gain-frequency response for a "non-tapered" structure. The plots for the nontapered structure (not presented here) would show that the bandwidth of the device is widened, though at the cost of gain, with a decrease in the waveguide-wall radius, disc-hole radius, and disc-thickness as well as with an increase in the disc-to-disc distance. Furthermore, such plots would also show that the midfrequency of the amplification band would appreciably change with a change in the disc-hole radius. Thus, one may choose the scheme of tapering the disc-hole radius according to (1c) to distribute the midfrequency of the amplification band over a wide range of frequencies, without regard to the gain value. In addition to the scheme of tapering the disc-hole radius, one may choose the scheme of tapering the waveguide-wall radius according to (1a)to compensate for the device gain that would be reduced at the band edges due to the tapering of the disc-hole radius. Hence, this has led us to propose the double tapering scheme involving simultaneous tapering of the waveguide-wall radius and the disc-hole radius according to (1a) and (1c), respectively, with due care to synchronously profile the magnetic flux density, and hence also the relevant beam parameters, as stated in the beginning of this section. As expected, suitable tapering of waveguide parameters widens the device bandwidth, though at the cost of gain, irrespective of whether the waveguide is smooth-walled or disc-loaded (Fig. 2). However, the proposed scheme of double tapering a disc-loaded waveguide has certainly predicted an enhancement of both the gain and bandwidth of the device as compared to the scheme of tapering the waveguide-wall radius of a smooth-wall waveguide, the mid-band frequency being adjusted, typically, around 40 GHz, by an appropriate selection of structure dimensions, namely the waveguide-wall radius, in the case of a smooth-wall waveguide, while the waveguide-wall and disc-hole radii, in the case of a disc-loaded waveguide (Fig. 2). A simple small-signal analysis has been used to predict the merit of the proposed scheme. It is however felt that the prediction would be more accurate, if a more rigorous nonlinear large-signal analysis, such as by Kurayeve *et al.* [19] and Kolosov and Kurayeve [20], were used.

IV. Conclusion

The results of beam-absent analysis of a disc-loaded circular waveguide fed into the gain-equation of a gyro-TWT gave a method to distribute the midband frequency of amplification over a wide range of frequencies by tapering the disc-hole radius, and at the same time compensate for gain reduction at band edges, which is caused by disc-hole radius tapering, by simultaneously tapering the waveguide-wall radius. Thus, the method of double-tapering the structure predicted a wide device bandwidth at an appreciable gain of a gyro-TWT.

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