Modal Analysis of a Corrugated Circular Waveguide for Wideband Gyro-Traveling-Wave Tubes

Vishal Kesari, P. K. Jain and B. N. Basu

Centre of Research in Microwave Tubes, Department of Electronics Engineering
Institute of Technology, Banaras Hindu University, Varanasi 221 005, India
Tel: 91-542-2322237; Fax: 91-0542-2366758; E-mail: vishal_kesari@rediffmail.com

***Abstract*- The field matching technique was used for the modal analysis of a circular waveguide, which is corrugated to form discs between corrugations. The dispersion relation of the structure has been derived considering all the harmonics of the traveling waves in the corrugation-free region and the stationary waves in the region of corrugation. The results have been validated against that reported earlier and also using HFSS, for azimuthally symmetric modes, with particular reference to the modes TE_{01}, TE_{02}, and TE_{03}. The mode TE_{01} and the axial periodicity of discs proved to be the most effective in controlling the dispersion characteristics of the structure for wideband performance of a gyro-TWT millimeter-wave amplifier.**

***Index Terms*- Axially periodic circular waveguide, field matching technique, gyro-traveling-wave tube, broadband millimeter-wave amplifier.**

I. INTRODUCTION

High-resolution and long range imaging radars and high-speed and high-information density communication systems need high-power broadband millimeter-wave amplifiers such as a gyro-traveling-wave tube (gyro-TWT). However, the gyro-TWT has a limited bandwidth due to the dispersion of the waveguide interaction structure of the device that is operated near the waveguide cutoff. This demands the techniques to reduce the structure dispersion for widening the gyro-TWT bandwidth. Two such techniques are dielectric loading [1] and corrugating the waveguide wall [2]. However, one should prefer corrugating the waveguide wall to loading it by a dielectric, as the latter poses the problem of dielectric charging and heating due to dielectric loss.

A circular waveguide radially corrugated with a regular axial periodicity, also referred to as a disc-loaded waveguide (Fig. 1), has proved its potential as the RF interaction structure of traveling-wave tube (TWT) amplifiers [3]-[5], backward-wave oscillators [6]-[8] and linear accelerators [4], [8]. It also found applications as electromagnetic filters [8]-[11], phase shifters [2], [10], [11], corrugated antennas [2], [10], antenna feeds [2], [9], etc. The disc-loaded waveguide has been analyzed by the simple surface impedance model [2], which is, however, valid for closely spaced corrugation or by an involved coupled integration technique [10] for Floquet's modes. The structure has also been analyzed by the field matching technique, for instance by Choe and Uhm [12] and Kesari *et al.* [13], considering traveling waves in the corrugation-free (or disc-free) region and stationary waves in the region of corrugation (or discs). In the analysis of Choe and Uhm [12], only the lowest order, stationary wave mode in the corrugation region and only the fundamental, traveling wave mode in the corrugation-free region, were considered, while, in the analysis of Kesari *et al.* [13], higher order harmonics were considered in both the corrugation-free and corrugation regions. However, for the sake of simplicity, both Choe and Uhm [12] and Kesari *et al.* [13] ignored the effect of the finite thickness of discs formed by corrugation. Further, they presented their results only for the lowest order azimuthally symmetric mode TE_{01}. In the present paper, the analysis of Kesari *et al*. [13], taking higher order harmonics in both the corrugation-free and corrugation regions, is generalized by considering all the axial dimensions of the corrugated structure to be finite (Fig. 1). This amounts to taking into account in the analysis the effect of finite disc thickness of a disc-loaded waveguide ignored by Choe and Uhm

[12] and Kesari *et al.* [13], and presenting the results not only for the lowest order TE_{01} mode but also for higher order azimuthally symmetric modes ignored by them. The dispersion relation of the corrugated circular waveguide (Section II) is used to study the effects of the corrugation parameters on the dispersion characteristics of the structure for the various modes (typically, TE_{01}, TE_{02}, and TE_{03}) (Section III).

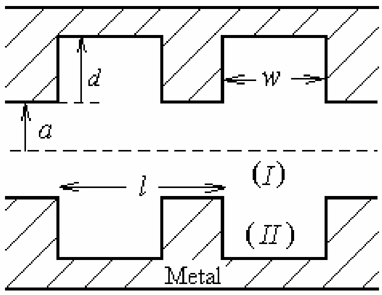


Fig.1. Schematic of the radially corrugated circular waveguide.

II. ANALYSIS

The corrugated circular waveguide (Fig. 1) is divided into two regions — the corrugation-free (or disc-free) region: $0 \le r < a$ and $0 < z < l$, labeled as region I, and the corrugation (or disc) region: $a \le r < (a+d)$ and $0 < z < w$, labeled as region II, where a, w, and d are respectively the inner edge radius, width and depth of corrugation, which may also be interpreted as the disc-hole radius, the axial gap between consecutive discs, and the radial disc thickness, respectively (Fig. 1).

The RF fields, for non-azimuthally varying $(\partial/\partial\theta = 0)$ TE modes $(E_z = 0)$, which are extensively used in a gyro-TWT amplifier, enjoying a significant growth rate at the operating frequency near the waveguide cutoff of a gyro-TWT amplifier [14], may be expressed in the cylindrical system of coordinates (r, θ, z), as [12], [13]:

$$H_z^I = \sum_{n=-\infty}^{\infty} A_n J_0\{\gamma_n r\} \exp j(\omega t - \beta_n z) \qquad (1)$$

$$E_\theta^I = j\omega\mu_0 \sum_{n=-\infty}^{\infty} \frac{A_n}{\gamma_n} J_0'\{\gamma_n r\} \exp j(\omega t - \beta_n z) \qquad (2)$$

and

$$H_z^{II} = \sum_{m=1}^{\infty} A_m Z_0\{\gamma_m r\} \exp(j\omega t) \sin(\beta_m z) \qquad (3)$$

$$E_\theta^{II} = j\omega\mu_0 \sum_{m=1}^{\infty} \frac{A_m}{\gamma_m} Z_0'\{\gamma_m r\} \exp(j\omega t) \sin(\beta_m^{II} z) \qquad (4)$$

where

$$Z_0\{\gamma_m r\} = [Y_0'\{\gamma_m (a+d)\} J_0\{\gamma_m r\} - J_0'\{\gamma_m (a+d)\} Y_0\{\gamma_m r\}]$$

$$Z_0'\{\gamma_m r\} = [Y_0'\{\gamma_m (a+d)\} J_0'\{\gamma_m r\} - J_0'\{\gamma_m (a+d)\} Y_0'\{\gamma_m r\}].$$

The superscripts I and II refer to the regions I and II, respectively. J_0 and Y_0 are the zeroth order Bessel functions of the first and second kinds, respectively, and the prime with these functions indicates their derivatives with respect to argument. Here, referring respectively to the two regions I and II, (i) A_n and A_m are the field constants; (ii) γ_n $(= (k^2 - \beta_n^2)^{1/2})$ and γ_m $(= (k^2 - \beta_m^2)^{1/2})$ are the radial propagation constants; and (iii) β_n $(= \beta_0 + 2\pi n/l)$ (traveling-wave) and β_m $(= m\pi/w)$ (stationary-wave) [13] are the axial phase propagation constants, k being the free-space propagation constant, and n $(= 0, \pm 1, \pm 2, ...)$ and m $(= 1, 2, 3, ...)$ representing the space and modal harmonic numbers, respectively. The absence of Bessel function of the second kind ($Y_0\{\gamma_m r\}$) in (1) as well as its derivative thereof ($Y_0'\{\gamma_m r\}$) in (2) conforms to the requirement that the fields do not reach to infinity at the structure axis (in view of the functions $Y_0\{x\}$ and $Y_0'\{x\}$ each $\to \infty$ as $x \to 0$).

The relevant boundary conditions referring to the continuity of the tangential magnetic (axial) and electric (azimuthal) field intensities at the interface between the regions I and II (at $r = a$) (Fig. 1) are respectively:

$$H_z^I = H_z^{II} \quad 0 < z < w \qquad (5)$$

$$E_\theta^I = \begin{cases} E_\theta^{II} & 0 < z < w \\ 0 & w < z < l \end{cases} . \qquad (6)$$

The field expressions (1)-(4) are substituted into the boundary conditions (5) and (6), each of which is then multiplied by $\sin(\beta_m z)$. Next, the resulting equation from (5) is integrated between $z = 0$ to *w* and that from (6) between $z = 0$ to *l* (Fig. 1). This would yield, using the orthogonal properties of trigonometric functions, two different series expressions in A_n each for A_m. Equating the right hand sides of these two series expressions, one obtains a series equation in A_n corresponding to a value of *m*. Thus, assigning different values of *m*, and following the method outlined in Keasri *et al.* [13], one may form *m* number of such series equations in A_n, the condition for non-trivial solution of which yields the dispersion relation of the structure as:

$$\det\left|(\xi_{nm} - \zeta_{nm})\right| = 0 \qquad (7)$$

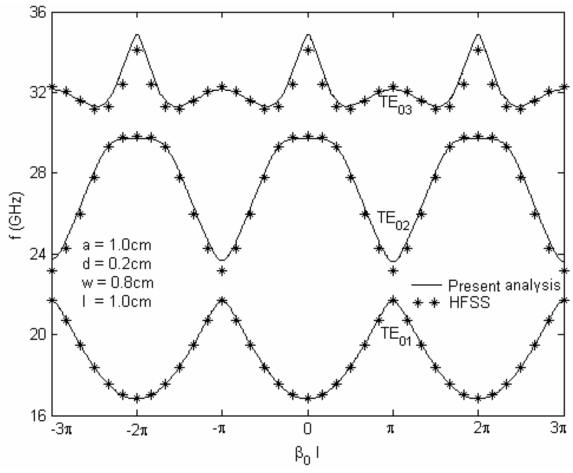
where

$$\xi_{nm} = \gamma_n \beta_m Z_0'\{\gamma_m a\} J_0\{\gamma_n a\} \times \times [1 - (-1)^m \exp(-j\beta_0 l)]$$

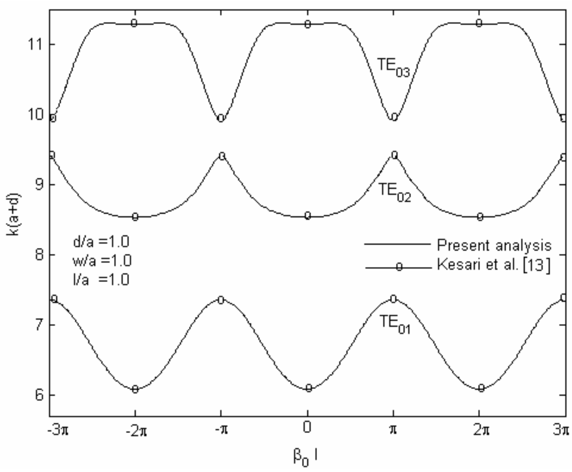
$$\zeta_{nm} = \gamma_m Z_0\{\gamma_m a\} J_0'\{\gamma_n a\}\big[\beta_m - \exp(-j\beta_0 l)\big(\beta_m \cos(\beta_m l) + j\beta_n \sin(\beta_m l)\big)\big].$$

III. RESULTS AND DISCUSSION

The dispersion relation (7) of the corrugated circular waveguide (Fig. 1) involves a determinant of infinite order and is not amenable to easy solution. However, for practical structure geometries, it is fair enough to take *n* = -2, -1, 0, 1, 2 and *m* = 1, 2, 3, 4, 5, or in other words truncate the determinant in (7) to 5×5 [13], for the desired converging solutions using numerical methods, in MATLAB. Hence the corrugated circular waveguide is studied for the effect of the corrugation parameters on the dispersion characteristics, with reference to the typical modes TE_{01}, TE_{02}, and TE_{03}, although the dispersion relation (7) is valid in general for all the azimuthally symmetric modes TE_{0n} ($1 \le n < \infty$) (Figs. 2 and 3).



(a)

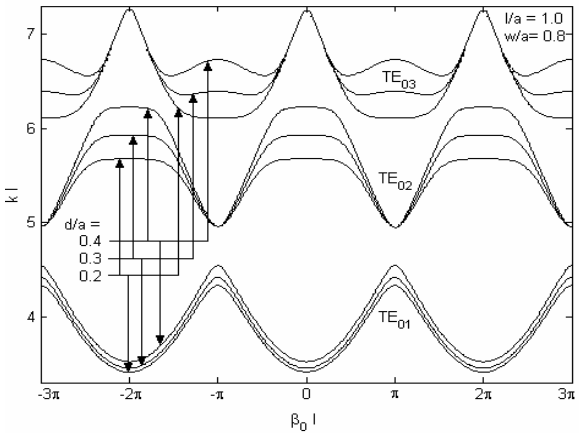


(b)

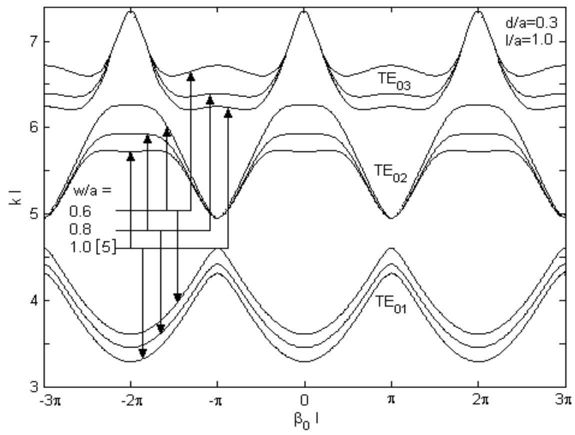
Fig.2. Dispersion characteristics of the corrugated circular waveguide obtained by the present analysis validated (a) against the simulation software — HFSS considering the axial structure dimensions to be finite and (b) against those of Kesaei *et al.* [13] considering discs of infinitesimally thin axial dimension. [The ordinate and abscissa are suitably adjusted for the sake of comparison with [13] in (b)].

It can be easily verified with the help of the dispersion relation (7) that the product of the wave number, corresponding to $\beta_0^I = 0$, and the inner edge radius *a* of corrugation, for the depth parameter $d/a = 0$ and the same product for the width parameter $w/a = 0$, with reference to the typical modes TE_{01}, TE_{02}, and TE_{03} considered, would become each identical with the corresponding product values for a smooth-wall circular waveguide for these modes. Furthermore, the dispersion characteristics obtained by the present analysis are within 0.1% and 1.5% of HFSS for the lowest order mode (TE_{01}) and higher order modes (TE_{02} and TE_{03}), respectively [Fig. 2(a)]. For the special case of $d \rightarrow 0$ (zero corrugation depth or radial disc thickness), as expected, the dispersion relation (7) passes on to that of a smooth-wall circular waveguide excited in the TE_{0n} mode. Also, the dispersion relation (7) becomes identical with that obtained by Choe and Uhm [12] for a circular waveguide loaded with thin discs for the special case of $w \rightarrow l$, ignoring higher order harmonics, that is, considering only the lowest order space and modal harmonics in the disc-free and disc-occupied regions, respectively. Furthermore, for infinitesimally thin discs ($w \rightarrow l$), but considering higher order harmonics in these two regions, the dispersion characteristics obtained with the help of (7) agree with those of Kesari *et al.* [13] [Fig. 2(b)].

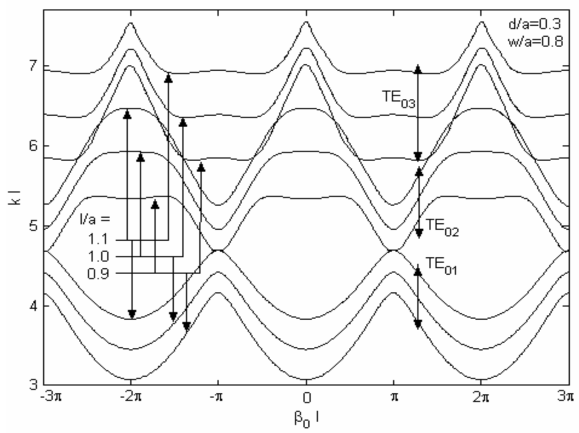
The circular waveguide, which is inherently a high-pass filter with a cutoff, if corrugated periodically, will exhibit a bandpass characteristics with alternate stop and pass bands on the frequency scale kl with their respective cutoffs, and a periodicity of 2π on the propagation constant scale $\beta_0 l$ (Figs. 2 and 3). This periodic nature of the dispersion plots is the result of frequency band splitting due to interaction of the waves travelling in the forward and backward directions that results into coupling of modes [3], [4], [8]. The lower and upper cutoff frequencies of a passband are the consequences of smooth-wall waveguide and axial periodicity of the structure, respectively. This is demonstrated for the typical structure parameters with reference to the three lowest order modes TE_{01}, TE_{02} and TE_{03}. In general, the structure being periodic in nature, exhibit positive, zero and negative slopes of the kl verses $\beta_0 l$ dispersion plots corresponding to positive, zero and negative group velocities, respectively, which can be adjusted by varying the structure parameters, namely, depth, width and periodicity parameters. This reveals that the group velocity can be easily adjusted by varying the structure parameters of a corrugated waveguide.



(a)



(b)



(c)

Fig.3. Dispersion characteristics of the corrugated circular waveguide taking the corrugation parameters, namely (a) depth (radial thickness of the disc), (b) width (axial gap between two consecutive discs), and (c) periodicity, all relative to the inner edge radius of corrugation, as the parameters, typically for the three modes TE_{01}, TE_{02}, and TE_{03}.

Further, since the propagation constant of two space harmonics corresponding to positive and negative values of n correspond to same frequency according to the Floquet's theorem, it follows the symmetric nature of dispersion characteristics (Figs. 2 and 3) around a phase shift of zero ($\beta_0 l = 0$) and π radian ($\beta_0 l = \pi$) of electromagnetic wave [3], [4], [8]. The existence of upper cutoff at $\beta_0 l = \pi$ (Figs. 2 and 3), typically for the lowest order mode, is due to the diverging nature of the sum of the harmonics of reflected waves. At this state, there will be a strong reflected wave and in the final state the equipartition of energy between the waves travelling in the forward and backward directions to setup a pure standing wave pattern with zero group velocity corresponding to zero slope of the $k l$ verses $\beta_0 l$ dispersion plots that represents no power flow in the structure (Fig. 1). Also, the estimate of upper cutoff frequency can be correlated with the resonance frequency of the hollow annular cavity [3], [4], [8].

The effects of the corrugation parameters on the dispersion control in general depends upon whether or not the frequencies are near the upper or lower edge frequencies of the passband (Fig. 3). The depth [Fig. 3(a)] as well as width [Fig. 3(b)] parameter has a better control on the structure dispersion at the upper edge frequency for the TE_{02} mode and at the lower edge frequency for the TE_{03} mode. The upper edge frequency for the TE_{02} mode increases with the depth parameter while it decreases with the width parameter. These parameters have somewhat a uniform but less control on the dispersion characteristics over the entire passband for the TE_{01} mode [Figs. 3(a) and 3(b)].

The shape of the dispersion characteristics of the structure is most sensitive to the periodicity parameter over the entire pass band, however the effect is more pronounced at the upper edge frequencies for the TE_{02} and at the lower edge frequencies for the TE_{01} and TE_{03} modes [Fig. 3(c)]. While the depth and width parameters are effective in controlling the width of the passband, the periodicity parameter is effective in shifting the passband on the frequency scale (Fig. 3). Also, out of the three corrugation parameters, the periodicity is the most effective parameter in controlling shape of the dispersion characteristics of the structure. Furthermore, the straight-line portion of the dispersion characteristics over which the beam-mode dispersion line could be coalesced, for wideband gyro-TWT performance [13], is obtained at lower values of the axial phase propagation constant β_0 for the TE_{01} mode than for the higher order modes (TE_{02} and TE_{03}) suggesting the preference of the TE_{01} mode over higher order modes from the standpoint of the reduced effect of beam velocity spread (β_0 close to zero) on the device performance (Fig. 3).

IV. CONCLUSION

Considering space harmonic effects due to the axial periodicity of the structure and taking into account the finiteness of axial structure dimensions as well as higher order waveguide modes have added practical relevance to the analysis of a corrugated circular waveguide for its potential use as a wideband interaction structure of a gyro-TWT amplifier. The shape of the dispersion characteristics of the corrugated circular waveguide has relevance to widening the bandwidth of coalescence between the beam-mode and waveguide-mode dispersion characteristics of a gyro-TWT and hence the bandwidth of the device. Clearly, out of the typical three modes (TE_{01}, TE_{02} and TE_{03}) considered, the lowest order mode TE_{01} is the most effective and, out of the three corrugation parameters, namely the depth, width and periodicity, the latter is the most effective in controlling, and more precisely, straightening the dispersion characteristics of the structure for the desired coalescence bandwidth and consequent wideband gyro-TWT performance. It is hoped that the present analysis of a corrugated circular waveguide in the absence of an electron beam (cold analysis) would be a useful feedback to the study of beam-wave interaction of a gyro-TWT millimeter-wave amplifier and its design for wideband performance.

ACKNOWLEDGMENT

One of the authors (Vishal Kesari) is thankful to the Centre of Advanced Study (University Grants Commission) for financial assistance.

REFERENCES

[1] S.J. Rao, P.K. Jain, and B.N. Basu, "Broadbanding of a gyro-TWT by dielectric-loading through dispersion shaping," *IEEE Trans. Electron Dev.*, Vol. 43, pp. 2290-2299, Dec. 1996.

[2] P.J.B. Clarricoats and A.D. Olver, *Corrugated Horns for Microwave Antennas*, Peter Peregrinus, London, 1984.

[3] R.E. Collin, *Foundation for Microwave Engineering*, McGraw-Hill, New York, 1988.

[4] D.A. Watkins, *Topics in Electromagnetic Theory*, John Wiley & Sons, New York, 1958.

[5] A. Rekiouak, B.R. Cheo, G. Wurthman, and C. Bates, "A slow wave structure for gyro-TWA H_{11} operation," *IEEE Trans. Microwave Theory Tech.*, Vol. 42, pp. 1091-1094, Jun. 1994.

[6] J. Benford and J. Swegle, *High-Power Microwaves*, Artech House, Boston, 1992.

[7] A.V. Gaponov-Grekhov and V.L. Granatstein, Ed., *Application of High Power Microwaves*, Artech House, Boston, 1994.

[8] E.L. Chu and W.W. Hansen, "The theory of disc-loaded waveguides," *J. Appl. Phys.* Vol. 18, pp. 996-1008, Nov. 1947.

[9] J. Uher, J. Bornemann, and U. Rosenberg, *Waveguides Components for Antenna Feed Systems. Theory and Cad*, Artech House, Norwood, 1993.

[10] S. Amari, R. Vahldieck, J. Bornemann, and P. Leuchtmann, "Spectrum of corrugated and periodically loaded waveguides from classical matrix eigenvalues," *IEEE Trans. Microwave Theory and Tech.*, Vol. 48, pp. 453-460, Mar. 2000.

[11] C. Rauscher, "Design of dielectric-filled cavity filters with ultrawide stopband characteristics," *IEEE Trans. Microwave Theory and Tech*, Vol. 53, pp. 1777-1786, May 2005.

[12] J.Y. Choe and H.S. Uhm, "Theory of gyrotron amplifiers in disc or helix-loaded waveguides," *Int. J. Electronics*, Vol. 53, pp. 729-741, Sep. 1982.

[13] V. Kesari, P.K. Jain, and B.N. Basu, "Approaches to the analysis of a disc loaded cylindrical waveguide for potential application in wide-band gyro-TWTs," *IEEE Trans. Plasma Sci.*, Vol. 32, pp. 2144-2151, Oct. 2004.

[14] A.W. Fliflet, "Linear and non-linear theory of the Doppler-shifted cyclotron resonance maser based on TE and TM waveguide modes," *Int. J. Electronics*, Vol. 61, pp. 1049-1080, Sep. 1986.