Asian Journal of Physics Vol. 17, No. 2 (2008)289-296

Electromagnetic analysis of ohmic quality factor for the tapered gyrotron cavity

Inderjit Singh, Umesh Kumar, Vishal Kesari[1], P. K. Jain[1], B. N. Basu[1]

School of Electrical Sciences, Vellore Institute of Technology, Vellore, India.

[1]Centre of Research in Microwave Tubes, Dept of Electronics Engg., Institute of Technology, Banaras Hindu University, Varanasi-205001, India.

A simple electromagnetic method has been presented, by dividing the input- and output-taper sections of cavity into numbers of non-tapered circular steps, to calculate the ohmic quality factor of tapered cavity of a gyrotron. The energy stored in cavity and power loss on cavity-wall, have been estimated in sinusoidal field profile. For the typically chosen non-tapered cavity, the calculated value of ohmic quality factor, in the present paper, was found within 4% of the results calculated using two of previously published expressions. The validation/ comparison of results against simulation software: MAGIC have been made and found within 3%. Further, the effect of structure dimensions, namely, length of mid-section of the cavity, input-taper angle and output-taper angle, on value of the ohmic quality factor for a tapered gyrotron cavity have been studied. It is found that the ohmic quality factor for a tapered gyrotron cavity, other parameters remaining constant, increases with decrease of length of mid-section and input-taper angle, and with increase of output-taper angle. Out of input- and output-taper angles, the latter was found more sensitive controlling parameter for the ohmic quality factor and hence frequency selectivity as well as ohmic heating in a tapered cavity. Furthermore, the contours of length of mid-section and ohmic quality factor were plotted to provide flexibility to the designer in optimising the structure parameters of tapered cavity of a gyrotron.

1. Introduction

In millimetre-wave frequency range, the high-power devices find applications in long-range and high resolution radars, high information density communication including deep space and specialised satellite communication, electronic warfare, missile tracking and guidance, astronomy, remote sensing, directed energy weaponry, advanced electron accelerators for high energy physics research, electron cyclotron resonance heating of plasma, industrial heating, material processing, space-debris, waste remediation, hyperthermia – cancer therapy, ozone generation, atmospheric purification, and many more [1-22]. Among all the fast-wave gyro-devices developed, namely, the gyrotron, the gyro-klystron, the gyro-backward wave oscillator, the gyro-travelling wave tube amplifier, etc., the gyrotron has found the most attention, particularly for its application in electron cyclotron resonance heating for fusion plasmas for production of electric power using controlled thermonuclear fusion and material processing [3-20].

Out of the five typical subassemblies of a gyrotron, i) the magnetron injection gun for the formation of electron beams in helical trajectories; ii) the magnetic focusing structure forcing electron beams to follow helical trajectories; iii) the interaction resonant cavity where beam-wave interaction takes place; iv) the collector to collect unspent electron beams; and v) the output window from which the RF output will come out; for the operating frequency selectivity the interaction resonant cavity is most responsible [5-19]. Also, for high-power handling, the wall-losses for interaction resonant cavity should be less, that has been fixed to a desirable limit of 2.0 kW/cm^2 [5, 19]. The ohmic quality factor is the parameter which directly gives the operating frequency selectivity and is inversely proportional to the power dissipated due to resistive wall-losses [19, 21, 22]. It is evident that in order to have better frequency selectivity and to limit the ohmic loss in a cavity to an acceptable value, it is favourable to increase ohmic quality factor that will give mechanical stability to cavity. Therefore, it is important to study the ohmic quality factor and its variation with structure dimensions.

Corresponding author :
e mail: pkjain_ms@hotmail.com (Dr P K Jain)

The objective of the present paper is to propose a simple electromagnetic method, to calculate the ohmic quality factor for tapered gyrotron cavity by dividing cavity into numbers of non-tapered circular steps (Section 2). Further, the steps to develop the numerical program for the derived expression for the ohmic quality factor have been elaborated; the results obtained using MATLAB have been discussed (Section 3), the validation/ comparison of the results with those obtained using commercially available MAGIC simulation software, have been made and major conclusions drawn including the limitations of the work (Section 4).

2. Analysis

The ohmic quality factor of a non-tapered circular cavity of radius r_W and length L is defined as [21, 22]

$$Q = \frac{\omega W}{P_L}, \tag{1}$$

where ω is the angular frequency; W is the electromagnetic energy stored; and P_L is the power dissipated through the wall of cavity.

The electromagnetic energy stored W in the typically chosen, TE_{03}-mode excited non-tapered circular cavity of radius r_W and length L may be given as [21, 22]

$$W = \pi \mu_0 \int_{r=0}^{r=r_W} \int_{z=0}^{z=L} \left(|H_z|^2 + |H_r|^2 \right) r \, dr \, dz , \tag{2}$$

where μ_0 is the permittivity of free space; and H_z and H_r are axial and radial components of magnetic field intensity given as [21, 22]

$$H_z = H_{0z} J_0\{\chi_{03} r / r_w\} \sin(\pi z / L) \tag{3}$$

and

$$H_r = j \frac{[(\omega/c)^2 - (\chi_{03}/r_w)^2]^{1/2}}{\chi_{03}/r_w} H_{0z} J_1\{\chi_{03} r / r_w\} \sin(\pi z / L). \tag{4}$$

Here, H_{0z} is the field constant; J_0 and J_1 are Bessel functions of zeroth and first order; χ_{03} (=10.173) is the third eigenvalue of J_0; and c is speed of light in free space. It is assumed that the field intensity follows the sinusoidal profile along axis of the cavity. The power dissipated P_L through the wall of cavity may be given as [21, 22]

$$P_L = r_W \pi \left(\frac{\omega \mu_0}{2\sigma} \right)^{1/2} \int_{z=0}^{z=L} |H_z|^2 \Big|_{r=r_W} dz , \tag{5}$$

where σ is the electrical conductivity of the waveguide material. Due to azimuthally symmetric mode consideration, the azimuthal magnetic field intensity component does not appear in Eqs (2) and (5).

The interaction resonant cavity of the gyrotron oscillator is an open-ended circular cavity consisting of non-tapered mid-section, along with input down-taper and output up-taper sections. The purpose of input down-taper and output up-taper sections is to stopping the RF to reach gun-end and to connecting the gyrotron to plumbing lines close to collector-end. Due to down-taper, the radius towards gun-end is below cutoff, and the RF reflects totally from down-taper; also, due to up-taper, increase in radius towards collector-end changes the impedance of transmission line that results into partial reflection from up-taper. The radius of mid-section is close to cutoff, and even small changes lead to strong reflection [5-19]. Thus, the reflections from both the ends of mid-section result into stationary-wave formation and an increase in stored energy for beam-wave interaction.

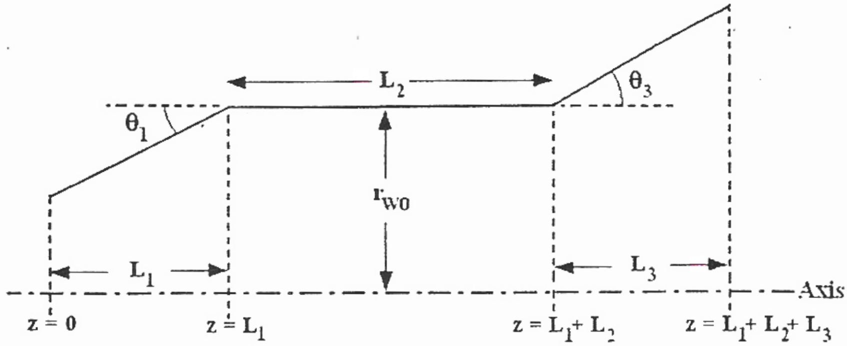


Fig. 1. Profile of the radius of a typical gyrotron resonant cavity including input- and output-taper.

Consider a gyrotron cavity of mid-section radius r_{W0} and length L_2, input-taper angle θ_1 and length L_1, and output-taper angle θ_3 and length L_3 (Fig. 1). (The subscript with the structure parameters represents the number of cavity section: 1 for input-taper section, 2 for non-tapered section, and 3 for output-taper section). Thus, the profile of resonant cavity radius may be given as

$$r = \begin{cases} r_{W0} - (L_1 - z)\tan\theta_1 & 0 < z < L_1 \\ r_{W0} & L_1 < z < L_1 + L_2 \\ r_{W0} + (z - L_1 - L_2)\tan\theta_3 & L_1 + L_2 < z < L_1 + L_2 + L_3 \,. \end{cases} \tag{6}$$

The gyrotron cavity may be approximated by the variation of radius over length of the structure in n number of steps, each step being non-tapered (constant radius). Therefore, the total energy stored in a gyrotron cavity may be summation of the energy stored in the individual steps as $W = W_1 + W_2 + W_3 + \ldots + W_n$, where W_n is the energy stored in n^{th} step of the stepped model of tapered gyrotron cavity. One may modify the expression (1) of ohmic quality factor of a non-tapered circular cavity for a gyrotron cavity as

$$Q = \frac{\omega}{P_L}(W_1 + W_2 + W_3 + \ldots\ldots + W_n) = \frac{\omega}{P_L}\sum_{q=1}^{n} W_q \,, \tag{7}$$

where W_q is the energy stored in q^{th} step of the stepped model of tapered gyrotron cavity; here, $q = 1$ and $q = n$ refer to the start and end steps of the structure, respectively (Fig. 1). The expression (7) for Q can be expressed in terms of the ohmic quality factor (Q_q for q^{th} step) and the power dissipated ($P_{L\,q}$ in q^{th} step) of the individual steps, as

$$Q = \frac{1}{P_L}\sum_{q=1}^{n} Q_q P_{L\,q} \,, \tag{8}$$

where Q_q can be straight forward written using Eq. (1) as

$$Q_q = \frac{\omega W_q}{P_{L\,q}} \tag{9}$$

Under the consideration of small taper angle, ratio of the power dissipated in q^{th} step to the power dissipated through the wall of cavity can be approximated to the ratio of power dissipated in q^{th} step to power dissipated through the wall of a virtual non-tapered circular cavity having radius same as that of q^{th} step and length same as that of tapered cavity, that is

$$\frac{P_{L\,q}(r = r_{W\,q})}{P_L} \approx \frac{P_{L\,q}(r = r_{W\,q})}{P_L(r = r_{W\,q})} \,. \tag{10}$$

where $P_L(r = r_{W\,q})$ is the power dissipated through the wall of a virtual non-tapered circular cavity having radius same as that of q^{th} step and length same as that of tapered cavity. Therefore, with the assumption of Eq. (10), Eq. (8) may be written as

$$Q = \sum_{q=1}^{n} Q_q \frac{P_{L\,q}(r = r_{W\,q})}{P_L(r = r_{W\,q})} . \qquad (11)$$

3. Results and Discussion

The Eq. (11) for the ohmic quality factor for a tapered gyrotron cavity involves a summation, in which each term corresponds to a step of the structure characterised by uniform radius. Each term of the summation contains three terms Q_q, $P_{L\,q}(r = r_{W\,q})$ and $P_L(r = r_{W\,q})$. Q_q can be calculated using the expression Eq. (9) in which W_q may be obtained substituting Eqs. (3) and (4) into Eq. (2); and $P_{L\,q}$ may be obtained substituting Eq. (3) into Eq. (5); where r_W should be replaced by $r_{W\,q}$; L in the limit of integration, by length of q^{th} step; and L in the sinusoidal function, by length of the cavity, so that the exact stored energy would be calculated following the sinusoidal field profile. $P_{L\,q}(r = r_{W\,q})$ is same as $P_{L\,q}$. However, $P_L(r = r_{W\,q})$ can be calculated substituting Eq. (3) into Eq. (5), where r_W should be replaced by $r_{W\,q}$, and both L, in the limit of integration and in the sinusoidal function, by length of the cavity. On the basis of the argument just discussed a numerical program is developed for a non-tapered circular cavity, and the summation has been performed following the radius profile defined as Eq. (6). Typically, here, the input- and output-taper sections are divided into hundred steps (n = 100), the result for ohmic quality factor so obtained (Q = 34700) for the non-tapered structure excited in TE_{031}-mode, for the typically chosen structure parameters, r_W (= 11.57 mm), σ (= 5.8×10^7 mhos/m (copper)), L (= 45 mm), has been compared with those Q = 34946 and 35944, obtained using the program developed taking the expression in [21] and [19], respectively, and the variations of ~ 1 % and ~ 4 % are found. Further, for typically chosen structure parameters (r_{W0} = 11.57 mm, σ = 5.8×10^7 mhos/m, L_1 = 30 mm, L_2 = 45 mm, L_3 = 46 mm, θ_1 = 2°, and θ_3 = 3°) for the TE_{031}-mode, a study for validation/ comparison of the results with respect to commercially available MAGIC simulation software has been made and presented (Table 1). It has been found that although with decrease of mesh size in MAGIC, the model of the cavity becomes smooth but time taken for computation of ohmic quality factor becomes larger and larger (Table 1).

Further, the effect of structure dimensions, namely, length of mid-section of the cavity, input-taper angle and output-taper angle, on value of the ohmic quality factor for a tapered gyrotron cavity have been studied. It is found that the ohmic quality factor for a tapered gyrotron cavity, in general, decreases with the increase of mid-section length of the cavity (upper four curves), however, it increases for greater values of output to input-taper angles (lower two curves) (Fig. 2). Thus, over the variation of mid-section length of the cavity, one can choose mid-section length as per the required Q for the desired ohmic heating.

Table 1. Ohmic quality factor obtained using present analysis and with simulation software: MAGIC (Taking typically tapered copper cavity with r_{W0} = 11.57 mm, L_1 = 30 mm, L_2 = 45 mm, L_3 = 46 mm, θ_1 = 2°, and θ_3 = 3°, and excited in the TE_{031}-mode.)

Analytical	MAGIC			
Q	Q	Mesh size (mm)	Computation time (minute)	~ % variation from analytical Q
	30024	0.50	20	17
36217	34251	0.25	53	5
	35135	0.15	135	3

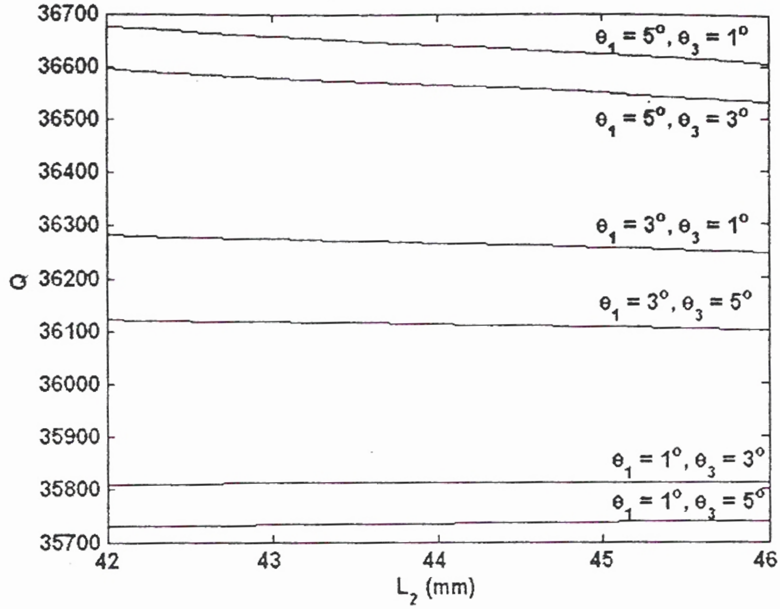
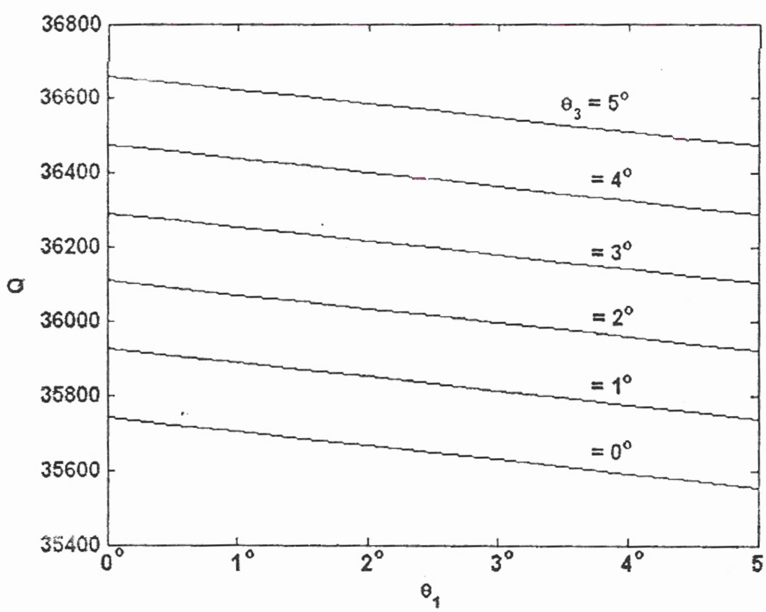
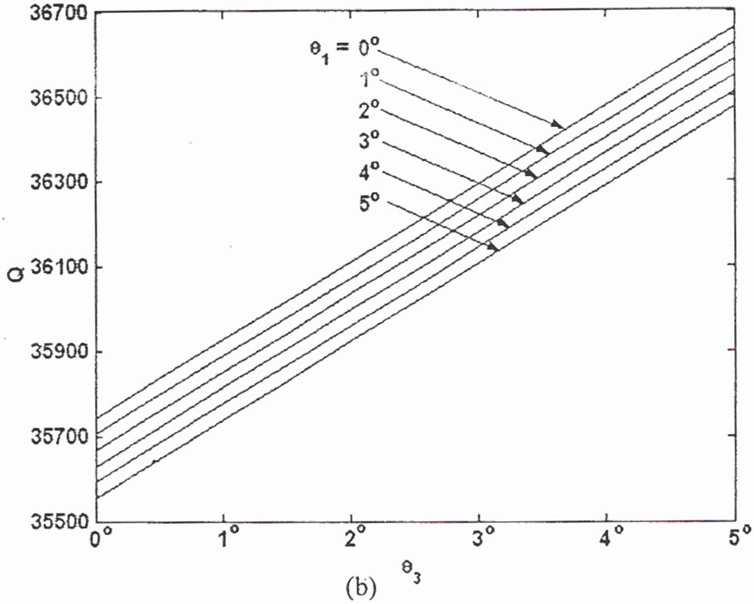


Fig. 2. Ohmic quality factor for a tapered gyrotron cavity verses mid-section length L_2 of the cavity taking input- and output-taper angles as the parameters (input-taper length L_1 = 30 mm, output-taper length L_3 = 46 mm).

For the constant value of output-taper angle, the value of ohmic quality factor for a tapered gyrotron cavity decreases with increase of input-taper angle, and for the constant value of input-taper angle, the value of ohmic quality factor for a tapered gyrotron cavity increases with increase of output-taper angle (Fig. 3). However, in the relative comparison of the ohmic quality factor with input- and output-taper angles, the latter is found more sensitive controlling parameter (Fig. 3) to control the value of ohmic quality factor, and hence the frequency selectivity as well as ohmic heating in the tapered gyrotron cavity, that itself would provide the higher power handling capacity to the cavity.

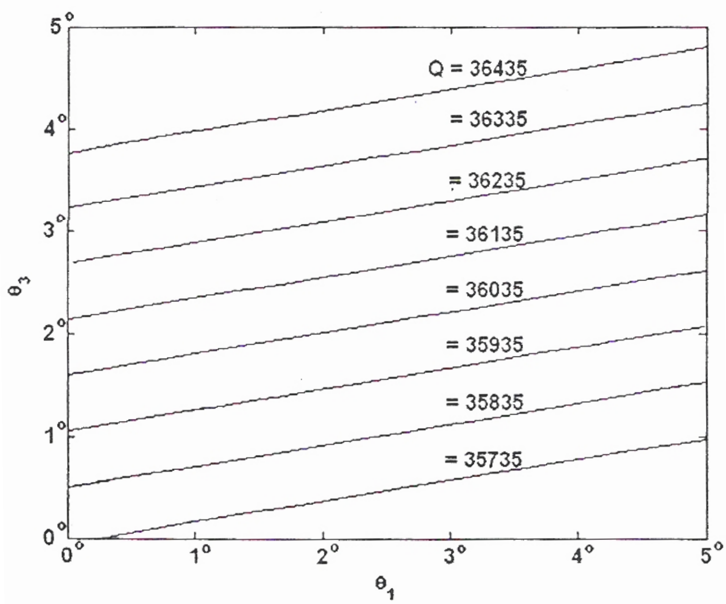


(a)

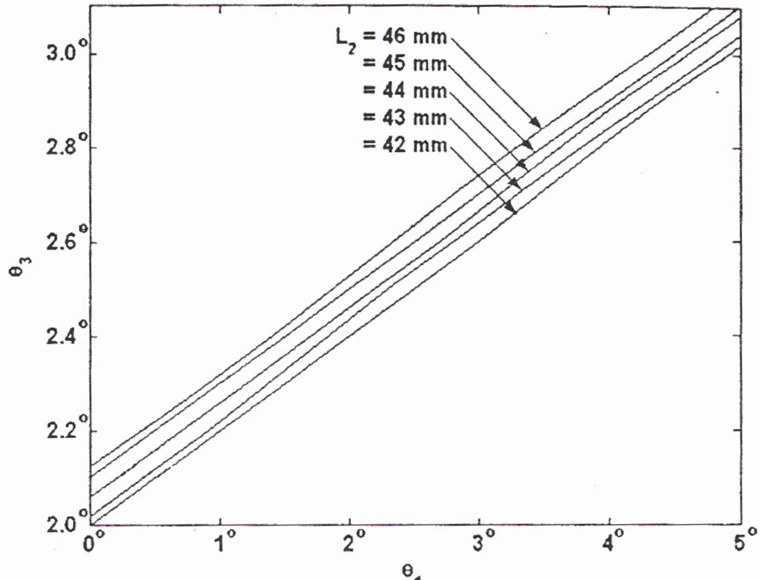


(b)

Fig. 3. Ohmic quality factor verses input-taper angle taking output-taper angle as a parameter (a) and ohmic quality factor verses output-taper angle taking input-taper angle as a parameter (b), for a tapered gyrotron cavity (radius of mid-section r_{W0} = 11.57 mm, and L_1 = 30 mm, L_2 = 45 mm, and L_3 = 46 mm).



(a)



(b)

Fig. 4. Ohmic quality factor contours to optimise ohmic heating for mid-section length $L_2 = 45$ mm (a), and mid-section contours to optimise mid-section length for $Q = 36125$ (b) in a tapered gyrotron cavity as a function of input- and output-taper angles (radius of mid-section $r_{W0} = 11.57$ mm, $L_1 = 30$ mm and $L_3 = 46$ mm).

Furthermore, for the optimisation of tapered gyrotron cavity, mid-section length contours are plotted for a set of values of ohmic quality factor in which optimisation have been made for the ohmic quality factor of cavity (Fig. 4a), and ohmic quality factor contours are plotted for a set of values of the mid-section length of cavity in which optimisation have been made for the mid-section length of cavity (Fig. 4b). Basically, plotted contours (Fig. 4) provide flexibility to the gyrotron cavity designer in optimising the input- and output-taper angles for constant value of the ohmic quality factor (Fig. 4a) and for constant value of the mid-section length of cavity (Fig. 4b).

4. Conclusion

In the present paper, a simple electromagnetic analysis has been presented to calculate the ohmic quality factor of a weakly tapered gyrotron cavity, taking the expressions for the non-tapered circular cavity and stepping the tapered cavity. Here, typically the input- and output-taper sections of cavity is divided into small steps, however, the numbers of steps can be made higher to obtain more accurate values, at the cost of larger computation time. The sinusoidal field profile is taken to calculate energy stored in the cavity and power loss on the wall of cavity, it would be more accurate to consider exact field profile as calculated in [5, 19], that includes the complex differential equation of normalised field profile. The comparison of results has been estimated against well-known simulation software: MAGIC. Although, the work has limitations of including higher values of input- and output-taper angles, it still promise to provide flexibility to the gyrotron designer in optimising the cavity parameters.

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[*Received* : 5.3.2007; *accepted* : 15.7.2007]