

Slope of a Line: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$

Point-Slope Equation of a Line: $y - y_1 = m(x - x_1)$

The Slope-Intercept Equation of a Line: $y = mx + b$

Definition of Tangent Line with Slope m : $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m = f'(x)$

Area of a Region under a Curve: $A = \int_a^b f(x) dx$

Mean Value Theorem of Integrals: $\int_a^b f(x) dx = f(c)(b - a)$

Average Value of a Function on an Interval: $f(c) = \frac{1}{b - a} \int_a^b f(x) dx$

Inverse Function: $f(g(x)) = x$, $x \in D(g)$ $g(f(x)) = x$, $x \in D(f)$

$f(x)$ and $g(x)$ are one-to-one functions $\rightarrow$ strictly monotonic on their entire domain

The Derivative of an Inverse Function: $g'(x) = \frac{1}{f'(g(x))}$, $f'(g(x)) \neq 0$

Area of a Region Between Two Curves: $A = \int_a^b [f(x) - g(x)] dx$, $f(x) \geq g(x)$ on $[a, b]$

The Volume of a Solid of Revolution:

The Disc Method: $V = \pi \int_a^b [R(x)]^2 dx$ (horizontal axis of revolution)

$V = \pi \int_c^d [R(y)]^2 dy$ (vertical axis of revolution)

The Washer Method: $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$

The Shell Method: $V = 2\pi \int_a^b r(x)h(x) dx$ (vertical axis of revolution)

$V = 2\pi \int_c^d r(y)h(y) dy$ (horizontal axis of revolution)

Arc Length: $s = \int_a^b \sqrt{1 + [f'(x)]^2} dx$ $s = \int_c^d \sqrt{1 + [g'(y)]^2} dy$

The Area of a Surface of Revolution: $S = 2\pi \int_a^b r(x) \sqrt{1 + [f'(x)]^2} dx$ $S = 2\pi \int_c^d r(y) \sqrt{1 + [g'(y)]^2} dy$

Integration by Parts: $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Trigonometric Integrals:

Sine to the odd power $\rightarrow$ convert to cosines

Cosine to the odd power $\rightarrow$ convert to sines

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Secant to the even power $\rightarrow$ convert to tangents

Tangent to the odd power $\rightarrow$ convert to secants

Tangent to the even power without secants $\rightarrow$ convert to secants

Secant to the odd power $\rightarrow$ integration by parts $u = \sec x$ $du/dx = \sec^2 x$

OR convert to sines and cosines

Trigonometric Substitution:

$$\sqrt{a^2 - u^2} = a \cos \theta \quad u = a \sin \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sqrt{a^2 + u^2} = a \sec \theta \quad u = a \tan \theta$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\sqrt{u^2 - a^2} = \pm a \tan \theta \quad u = a \sec \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

The Trapezoidal Rule:

$$\int_a^b f(x)dx \approx \Delta x \left[\frac{1}{2} f(x_0) + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + \frac{1}{2} f(x_n) \right]$$

$$E \leq \frac{(b-a)^3}{12n^2} [\max |f''(x)|] \quad a \leq x \leq b$$

The Simpson's Rule (n is even):

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)]$$

$$E \leq \frac{(b-a)^5}{180n^4} [\max |f^{(4)}(x)|] \quad a \leq x \leq b$$

L'Hôpital's Rule: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \quad \frac{0}{0}, \frac{\infty}{\infty}$

Indeterminate Forms: $\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^\infty, \infty^0, 0^0, \infty - \infty$

Determinate Forms: $\infty + \infty \rightarrow \infty, -\infty - \infty \rightarrow -\infty, 0^\infty \rightarrow 0, 0^{-\infty} \rightarrow \infty$