DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\sin u] = (\cos u)u'$
12. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
13. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
14. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
15. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
16. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
17. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
18. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
19. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
20. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$
21. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
22. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int \sin u\,du = -\cos u + C$
8. $\int \cos u\,du = \sin u + C$
9. $\int \tan u\,du = -\ln|\cos u| + C$
10. $\int \cot u\,du = \ln|\sin u| + C$
11. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
12. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
13. $\int \sec^2 u\,du = \tan u + C$
14. $\int \csc^2 u\,du = -\cot u + C$
15. $\int \sec u \tan u\,du = \sec u + C$
16. $\int \csc u \cot u\,du = -\csc u + C$
17. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$
18. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
19. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

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Differentiation and Integration

The Limit Definition:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The Constant Rule: $c' = 0$

The General Power Rule: $(u^n)' = n \cdot u^{n-1} \cdot u'$

The Constant Multiple Rule: $(c \cdot u)' = c \cdot u'$

The Sum and Difference Rule: $(u \pm v)' = u' \pm v'$

The Product Rule: $(u \cdot v)' = u' \cdot v + u \cdot v'$

The Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}, \qquad v \neq 0$

The Chain Rule: $\frac{d}{dx}[f(u)] = f'(u) \cdot u'$

Implicit vs. Explicit Differentiation:
use implicit differentiation whenever it is difficult or impossible to solve an equation for “y”.