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Graph Sketching

The Original Function:

1. Domain (continuity): all inputs

   Common restrictions:

   a. Division by zero: $y = \frac{f(x)}{g(x)}$, $g(x) \neq 0$
   b. Even roots: $y = \sqrt[n]{f(x)}$, n-even number, $f(x) \geq 0$
   c. Trig functions:
      i. $y = \sin x$: $(-\infty, \infty)$
      ii. $y = \cos x$: $(-\infty, \infty)$
      iii. $y = \tan x$: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} + \pi n$
      iv. $y = \cot x$: $(-\infty, \infty)$ except for $x = \pi n$
      v. $y = \sec x$: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} + \pi n$
      vi. $y = \csc x$: $(-\infty, \infty)$ except for $x = \pi n$
   d. Log functions $y = \log_a x$: $x \in (0, \infty)$ and $a \in (0,1) \bigcup (1, \infty)$
   e. Exponential functions $y = a^x$: $x \in (-\infty, \infty)$ and $a \in (0,1) \bigcup (1, \infty)$

2. Range: all outputs

   Common functions:

   a. A line $y = mx + b$: $(-\infty, \infty)$
   b. A parabola $y = ax^2 + bx + c$: if $a>0$ then $[-\frac{b}{2a}, \infty)$

      if $a<0$ then $(-\infty, -\frac{b}{2a}]$
   c. Irrational functions $y = \frac{p(x)}{q(x)}$: $(-\infty, \infty)$ except for $\lim_{x \to \infty} \frac{p(x)}{q(x)}$ (HA) and removable discontinuities
   d. Even roots: $y = \sqrt[n]{f(x)}$, n-even number: $[0, \infty)$
   e. Trig functions:
      i. $y = \sin x$: $[-1,1]$
      ii. $y = \cos x$: $[-1,1]$
      iii. $y = \tan x$: $(-\infty, \infty)$
      iv. $y = \cot x$: $(-\infty, \infty)$
      v. $y = \sec x$: $(-\infty, -1] \bigcup [1, \infty)$
      vi. $y = \csc x$: $(-\infty, -1] \bigcup [1, \infty)$
   f. Log functions $y = \log_a x$: $(-\infty, \infty)$
   g. Exponential functions $y = a^x$: $(0, \infty)$

3. HA and VA
   HA: $y = \lim_{x \to \infty} f(x)$
   VA: if $y = \dfrac{p(x)}{q(x)}$ then $q(x) = 0$, but $p(x) \neq 0$
4. Intercepts
   y-intercepts: $f(0)$ $(0,y_0)$
   x-intercepts: $f(x)=0$ $(x_0, 0)$
5. Signs of function: $f(x) > 0$ and $f(x) < 0$

The First Derivative:

1. Increasing/decreasing
   $f(x)$ increases if f`(x)>0
   $f(x)$ decreases if f`(x)<0
2. Extrema (first/second derivative tests)

   The First Derivative Test for Extrema:
   1) Find the first derivative and set it equal to zero: $f'(x) = 0$
   2) Find the critical numbers when the equation equals to zero or undefined
   3) x_0 is a maximum of a function if at this point the sign of the derivative changes from positive to negative.
      x_0 is a minimum of a function if at this point the sign of the derivative changes from negative to positive.
   4) Find corresponding y-value: $y_0 = f(x_0)$ (x_0,y_0)

   The Second Derivative Test for Extrema:
   1) Find the first derivative and set it equal to zero: $f'(x) = 0$
   2) Find the critical numbers when the equation equals to zero or undefined
   3) Find the second derivative and evaluate it at x_0

      $f''(x_o)$ →
      $f''(x_o) > 0 \Rightarrow x_0$ is a min
      $f''(x_o) < 0 \Rightarrow x_0$ is a max
      $f''(x_o) = 0 \Rightarrow$ the test fails (use the first derivative test)
   4) Find corresponding y-value: $y_0 = f(x_0)$ (x_0,y_0)

The Second Derivative:

1. Inflection points: $f''(x) = 0$ or undefined
2. Concavity:
   $f''(x_0) > 0 \Rightarrow$ Concave upwards
   $f''(x_0) < 0 \Rightarrow$ Concave downwards