

## Graph Sketching

The Original Function:

1. Domain (continuity): all inputs

Common restrictions:

- a. Division by zero:  $y = \frac{f(x)}{g(x)}$ ,  $g(x) \neq 0$
- b. Even roots:  $y = \sqrt[n]{f(x)}$ , n-even number,  $f(x) \geq 0$
- c. Trig functions:
  - i.  $y = \sin x : (-\infty, \infty)$
  - ii.  $y = \cos x : (-\infty, \infty)$
  - iii.  $y = \tan x : (-\infty, \infty)$  except for  $x = \frac{\pi}{2} + \pi n$
  - iv.  $y = \cot x : (-\infty, \infty)$  except for  $x = \pi n$
  - v.  $y = \sec x : (-\infty, \infty)$  except for  $x = \frac{\pi}{2} + \pi n$
  - vi.  $y = \csc x : (-\infty, \infty)$  except for  $x = \pi n$
- d. Log functions  $y = \log_a x : x \in (0, \infty)$  and  $a \in (0, 1) \cup (1, \infty)$
- e. Exponential functions  $y = a^x : x \in (-\infty, \infty)$  and  $a \in (0, 1) \cup (1, \infty)$

2. Range: all outputs

Common functions:

- a. A line  $y = mx + b : (-\infty, \infty)$
- b. A parabola  $y = ax^2 + bx + c :$  if  $a > 0$  then  $[-\frac{b}{2a}, \infty)$   
if  $a < 0$  then  $(-\infty, -\frac{b}{2a}]$
- c. Irrational functions  $y = \frac{p(x)}{q(x)} : (-\infty, \infty)$  except for  $\lim_{x \rightarrow \infty} \frac{p(x)}{q(x)}$  (HA) and removable discontinuities
- d. Even roots:  $y = \sqrt[n]{f(x)}$ , n-even number:  $[0, \infty)$
- e. Trig functions:
  - i.  $y = \sin x : [-1, 1]$
  - ii.  $y = \cos x : [-1, 1]$
  - iii.  $y = \tan x : (-\infty, \infty)$
  - iv.  $y = \cot x : (-\infty, \infty)$
  - v.  $y = \sec x : (-\infty, -1] \cup [1, \infty)$
  - vi.  $y = \csc x : (-\infty, -1] \cup [1, \infty)$
- f. Log functions  $y = \log_a x : (-\infty, \infty)$
- g. Exponential functions  $y = a^x : (0, \infty)$

## 3. HA and VA

HA:  $y = \lim_{x \rightarrow \infty} f(x)$ VA: if  $y = \frac{p(x)}{q(x)}$  then  $q(x) = 0$ , but  $p(x) \neq 0$ 

## 4. Intercepts

y-intercepts:  $f(0) \quad (0, y_0)$ x-intercepts:  $f(x) = 0 \quad (x_0, 0)$ 5. Signs of function:  $f(x) > 0$  and  $f(x) < 0$ 

The First Derivative:

## 1. Increasing/decreasing

 $f(x)$  increases if  $f'(x) > 0$  $f(x)$  decreases if  $f'(x) < 0$ 

## 2. Extrema (first/second derivative tests)

The First Derivative Test for Extrema:

1) Find the first derivative and set it equal to zero:  $f'(x) = 0$ 

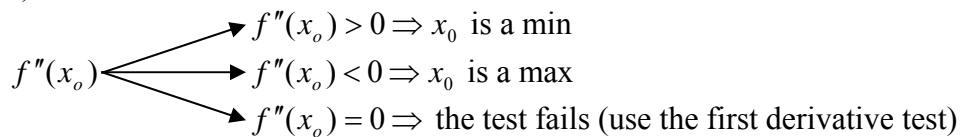
2) Find the critical numbers when the equation equals to zero or undefined

3)  $x_0$  is a maximum of a function if at this point the sign of the derivative changes from positive to negative. $x_0$  is a minimum of a function if at this point the sign of the derivative changes from negative to positive.4) Find corresponding y-value:  $y_0 = f(x_0) \quad (x_0, y_0)$ 

The Second Derivative Test for Extrema:

1) Find the first derivative and set it equal to zero:  $f'(x) = 0$ 

2) Find the critical numbers when the equation equals to zero or undefined

3) Find the second derivative and evaluate it at  $x_0$ 4) Find corresponding y-value:  $y_0 = f(x_0) \quad (x_0, y_0)$ 

The Second Derivative:

1. Inflection points:  $f''(x) = 0$  or undefined

## 2. Concavity:

 $f''(x_0) > 0 \Rightarrow$  Concave upwards $f''(x_0) < 0 \Rightarrow$  Concave downwards