VLADIMIR VORONOV

Tangent Lines

Equation of a line:
y=mx+b, *m* is the slope and *b* is the y-intercept (slope-intercept form)
$y-y_1=m(x-x_1)$, *m* is the slope, *y* and *x* are variables, and y_1 and x_1 is a point on the line (x_1,y_1) (point-slope form)

Find the equation of a line tangent to the function $f(x)$ at x_0:

1. Find the slope: $f'(x_0)=m$
2. Find the corresponding y-value: $f(x_0)=y_0$
3. Plug numbers into the point-slope form and solve for *y*:

$$y-y_0=m(x-x_0) \qquad \Rightarrow \qquad y=mx-mx_0+y_0$$

Example:
Find the equation of a line tangent to the function $f(x)=x^3+2x-6$ at x=1:

1. $f'(x)=3x^2+2$ $\qquad f'(1)=3(1)^2+2=5$
2. $f(1)=1^3+2\cdot 1-6=-3$
3. $y-(-3)=5(x-1) \qquad \Rightarrow \qquad y=5x-8$

Parallel lines have the same slope: $m_1=m_2$
Find the equation of a line tangent to the function $f(x)$ and parallel to the line $y=m_1x+a$:

1. Find the first derivative and set it equal to m_1 to find x_0: $f'(x_0)=m_1$
2. Find the corresponding y-value: $f(x_0)=y_0$
3. Plug numbers into the point-slope form and solve for *y*:

$$y-y_0=m_1(x-x_0) \qquad \Rightarrow \qquad y=m_1x-m_1x_0+y_0$$

Example:
Find the equation of a line tangent to the function $f(x)=x^3+2x-6$ and parallel to the line $y=5x+1$

1. $f'(x)=3x^2+2=5 \qquad \Rightarrow \qquad x=\pm 1$
2. $f(1)=1^3+2\cdot 1-6=-3$ $\qquad f(-1)=(-1)^3+2\cdot(-1)-6=-9$
3. $y-(-3)=5(x-1) \qquad \Rightarrow \qquad y=5x-8$
   $y-(-9)=5(x-1) \qquad \Rightarrow \qquad y=5x-14$

Perpendicular lines have slopes $m_1 \cdot m_2 = -1$ or $m_1 = -\frac{1}{m_2}$

Find the equation of a line tangent to the function *f*(*x*) and perpendicular to the line $y=m_2x+a$:

1. Find the slope $m_1 = -\frac{1}{m_2}$
2. Find the first derivative and set it equal to m_1 to find x_0: $f'(x_0) = m_1$
3. Find the corresponding y-value: $f(x_0) = y_0$
4. Plug numbers into the point-slope form and solve for *y*:

$$y - y_0 = m_1(x - x_0) \qquad \Rightarrow \qquad y = m_1x - m_1x_0 + y_0$$

Example

Find the equation of a line tangent to the function $f(x) = x^3 + 2x - 6$ and perpendicular to the line $y = -\frac{1}{5}x + 1$:

1. $m_1 = -\frac{1}{-1/5} = 5$
2. $f'(x) = 3x^2 + 2 = 5 \quad \Rightarrow \quad x = \pm 1$
3. $f(1) = 1^3 + 2 \cdot 1 - 6 = -3 \qquad f(-1) = (-1)^3 + 2 \cdot (-1) - 6 = -9$
4. $y - (-3) = 5(x - 1) \quad \Rightarrow \quad y = 5x - 8$
   $y - (-9) = 5(x - 1) \quad \Rightarrow \quad y = 5x - 14$