Resolution Enhancement of a Left-Handed Material Superlens

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Outline

An insight ahead of the time

SOVIET PHYSICS USPEKHI VOLUME 10, NUMBER 4 JANUARY-FEBRUARY 1968

538.30

THE ELECTRODYNAMICS OF SUBSTANCES WITH SIMULTANEOUSLY NEGATIVE VALUES OF ϵ AND μ

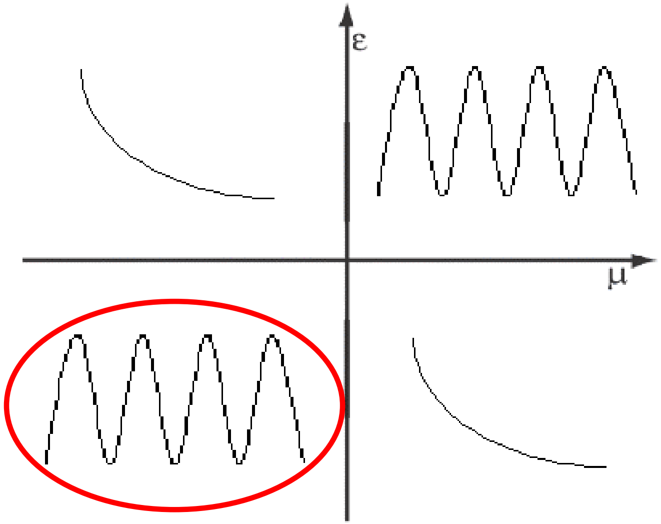
V. G. VESELAGO

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Usp. Fiz. Nauk 92, 517–526 (July, 1964)

$$\nabla^2 \vec{E} - \varepsilon\varepsilon_0 \mu\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\vec{E}(\omega) = \vec{E}_0 e^{j(\omega t - \vec{k}\cdot\vec{r})}, \quad \vec{k}^2 = \varepsilon\mu \frac{\omega^2}{c_0}$$



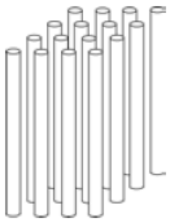
$$\varepsilon < 0, \mu < 0$$

Veselago's results

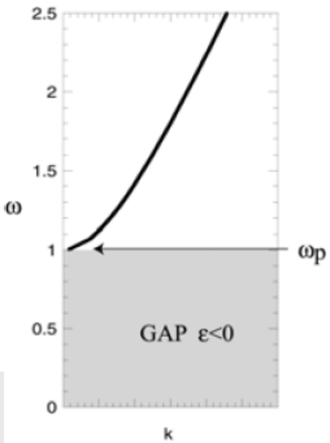
- ➢ Fundamental physical laws do not rule out the possibility of simultaneous negative ε and μ.
- ➢ EM properties of media with simultaneous negative ε and μ are very different from those of the normal materials with positive ε and μ.

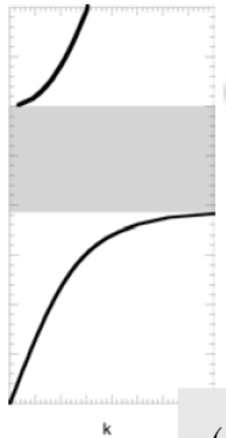
  - ✓ left-handed
  - ✓ negative index-of-refraction
  - ✓ reversed Doppler effect
  - ✓ reversed Cerenkov radiation
  - ✓ focusing through a slab of such a medium

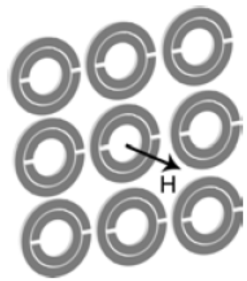
The rise of left-handed metamaterials



$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$







$$\mu(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2}$$

J. B. Pendry

PRL **76**,4773 (1996);

IEEE MTT **47**, 2075 (1999).

UCSD,

PRL **84**,4184 (2000);
Science **292**, 77 (2001).

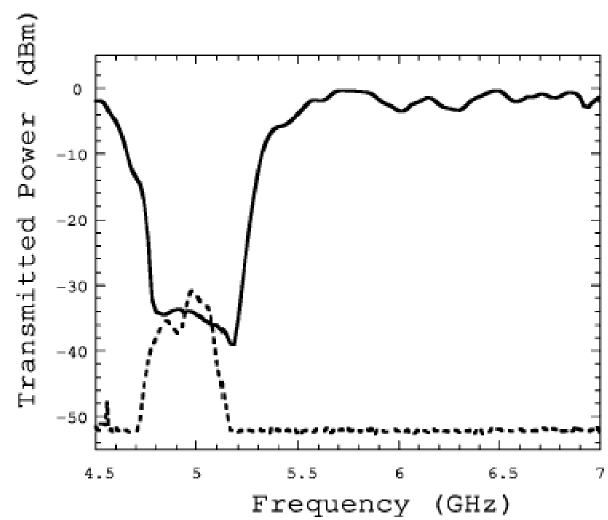
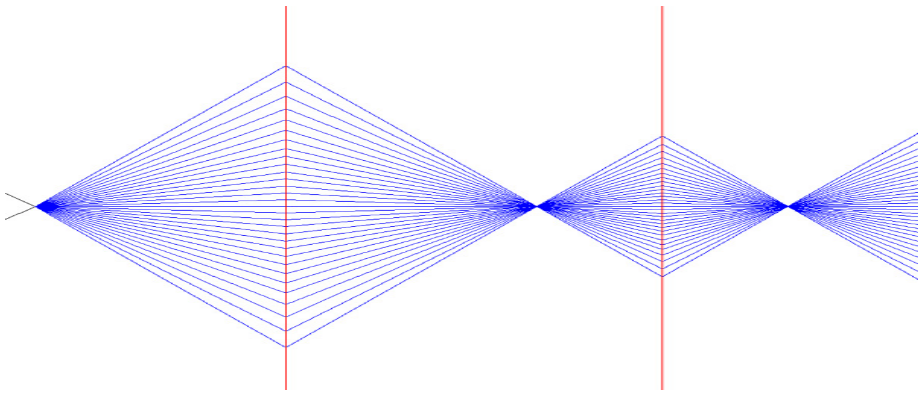


FIG. 3. A transmission experiment for the case of $H_\|$. The upper curve (solid line) is that of the SRR array with lattice parameter $a = 8.0$ mm. By adding wires uniformly between split rings, a passband occurs where μ and ε are both negative (dashed curve). The transmitted power of the wires alone is coincident with that of the instrumental noise floor (-52 dB).

Imaging by a LHM slab



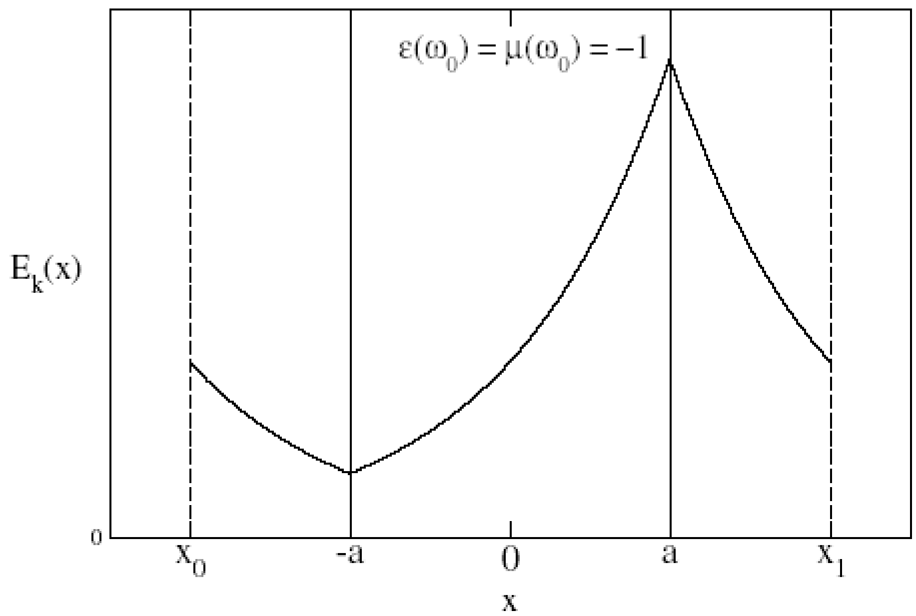
Propagating wave : Phase compensation (Veselago 1964)

Evanescent wave : Amplitude reconstruction (Pendry 2000)

"Perfect" lens $\varepsilon_r(\omega) = \mu_r(\omega) = -1$

All geometric details of the source can be reconstructed at the image!

Amplification of evanescent waves ?



No !

- Unphysically large field at the 2nd interface
- Breaking square integrability of EM fields if *t>d*
- Loss transforms amplified wave into decaying one
- … …

Methodology

Finite-difference time-domain (FDTD)

- ✓ full wave
- ✓ straight-forward
- ✓ causality guaranteed
- ✓ dynamical

Diverse FDTD results

- Ziolkovski & Heyman [*PRE* **64**, 056625 (2001)]

  No stable image observed

- Loschialpo *et al.* [*PRE* **67**, 025602 (2003)]

  stable image but no resolution enhancement observed

- Cummer [*APL* **82**, 1503 (2003)]

  image contains subwavelength components, enhanced resolution

- Karkkainen [cond-mat/0302407 (2003)]

  amplification of evanescent modes in a lossless LHM slab

Normal FDTD simulation involves both propagating and evanescent waves.

- Difficult to differentiate effects from propagating and evanescent wave.
- Difficult to study the dependence of the behavior of evanescent wave on different parameters.

How to generate *pure* evanescent wave ?

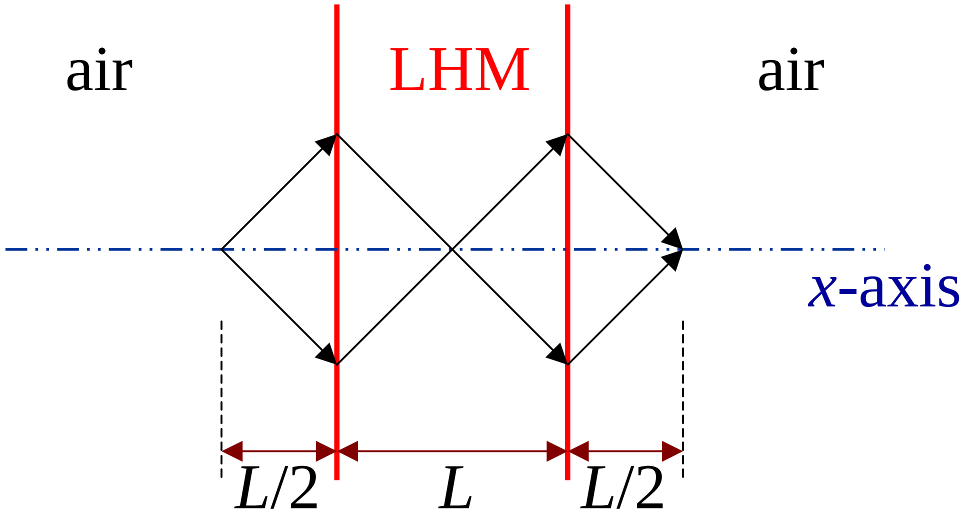
- Total internal reflection
- Guided mode of a planar dielectric waveguild
- "Periodic" boundary condition

$$\mathbf{E}(x,y,t)=\mathbf{E}_0 e^{-i(k_x x+k_y y-\omega t)}, \mathbf{H}(x,y,t)=\mathbf{H}_0 e^{-i(k_x x+k_y y-\omega t)}$$

$$k_x^2=\epsilon\mu\frac{\omega^2}{c^2}-k_y^2$$

plasmonic dispersion

$$\epsilon(\omega)=\mu(\omega)=1-\frac{\omega_p^2}{\omega^2-i\omega\nu_c}$$



z-polarized wave propagates in the *x*-direction

$$E_z(x,y,t)=E_{z0}e^{-i(k_x x+k_y y-\omega t)}$$

$$k_x^2+k_y^2=\frac{\omega^2}{c^2}$$

periodic boundary conditions are applied in the transverse *y*-direction

$$E_z(x,y\pm\Delta y)=E_z(x,y)e^{\mp i k_y \Delta y}$$ ($k_y^2>k_0^2$ for evanescent waves)

absorbing boundary conditions are applied at both ends of the *x*-direction

FDTD details

Yee cell with leapfrog staggered ***E*** and ***H*** sublattice

spatial grid $\Delta x = \Delta y = 0.3\ \text{mm}$

total simulation space $4000\Delta x \times 1\Delta y$

time step $\Delta t = \Delta x/(2c) = 0.5\ \text{ps}$

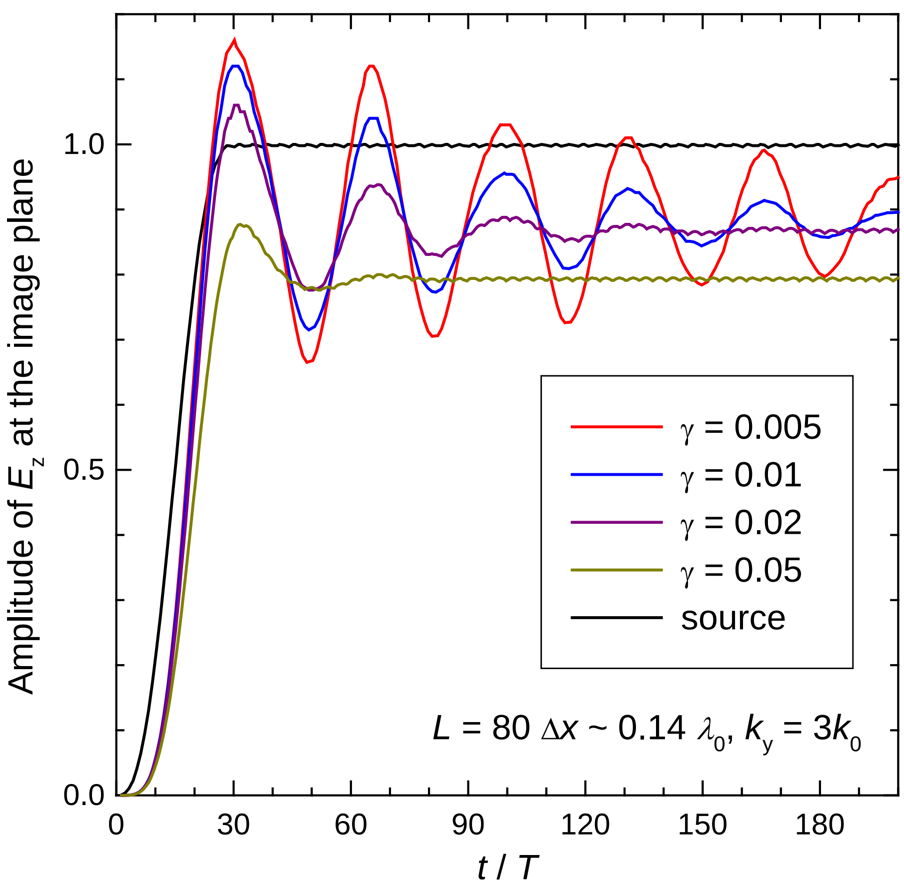
monochromatic EM source [Ziolkovski & Heyman, Phys. Rev. E 64, 056625 (2001)]

$$\omega_0/(2c) \approx 11\ \text{GHz} \qquad \lambda_0 \approx 566\Delta x$$

$$\epsilon(\omega_0) = \mu(\omega_0) = -1 - i\gamma$$

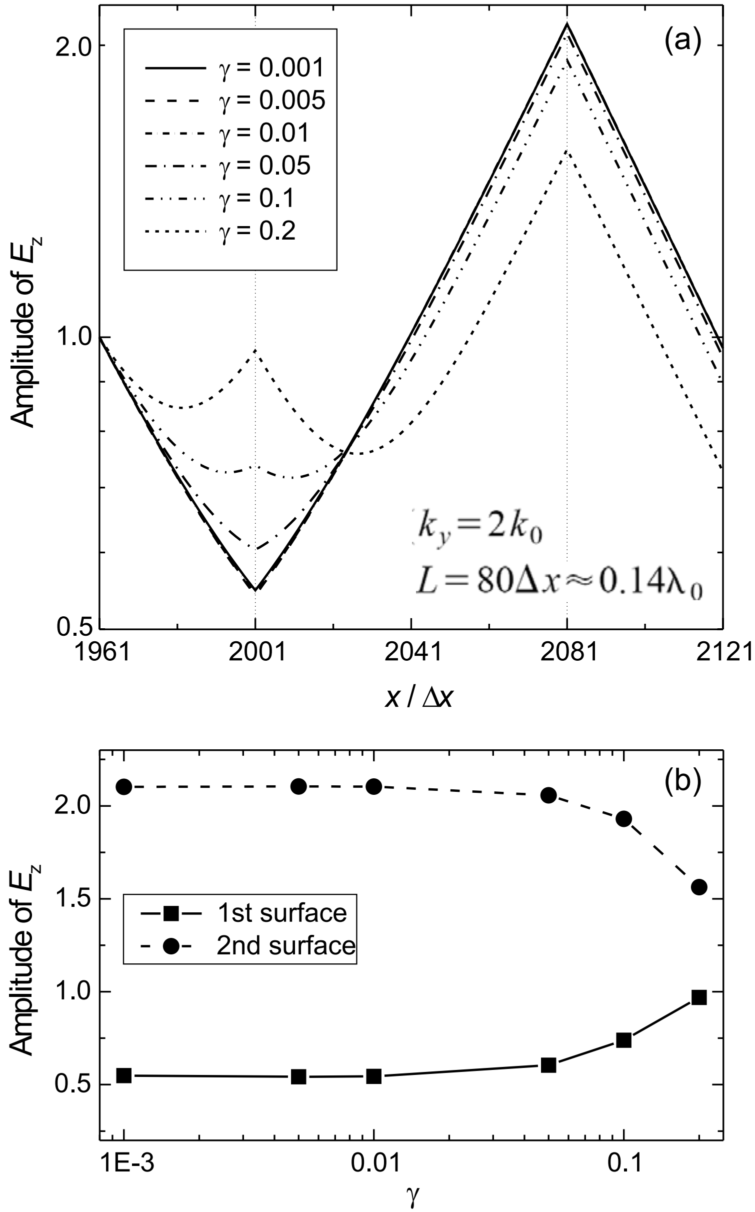
The dispersive ε and μ are handled in time domain using the piecewise-linear recursive convolution (PLRC) method

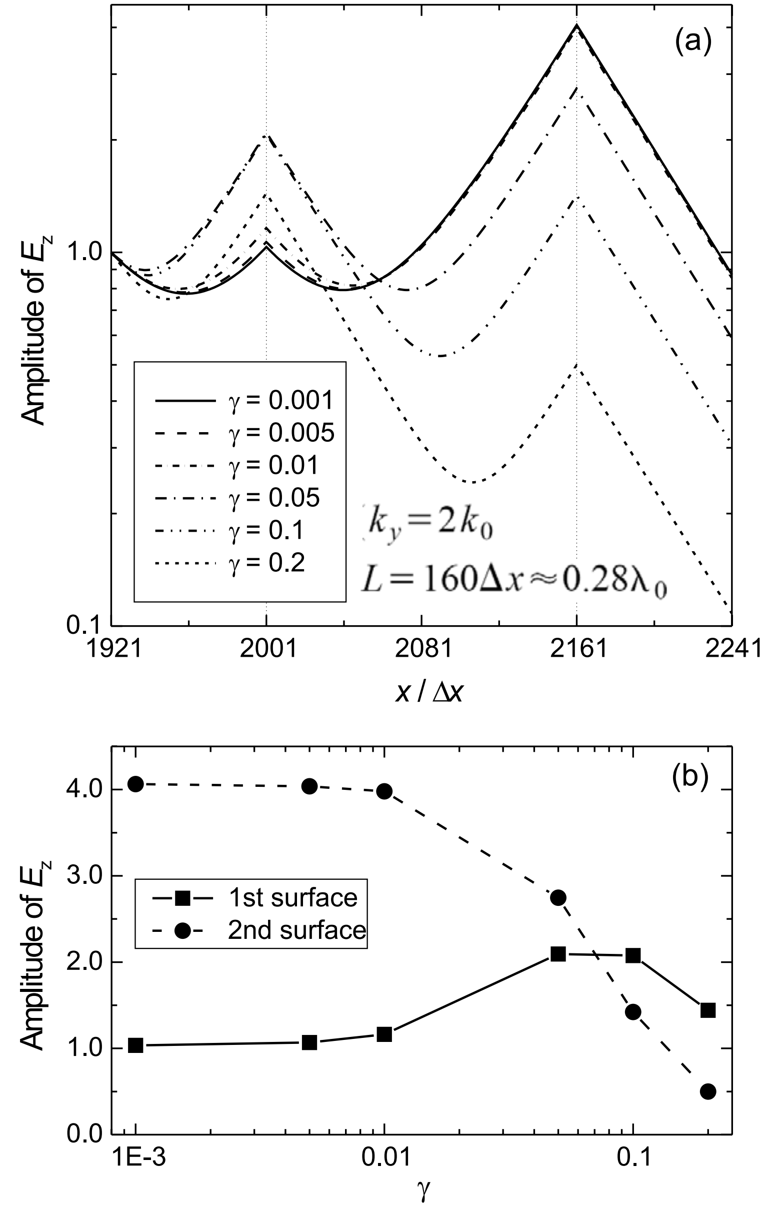
Dynamic features



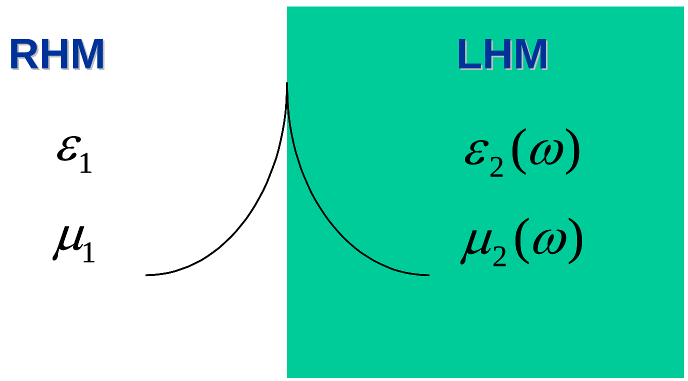
small absorption (or large transverse k) ➔ long relaxation time to stable state

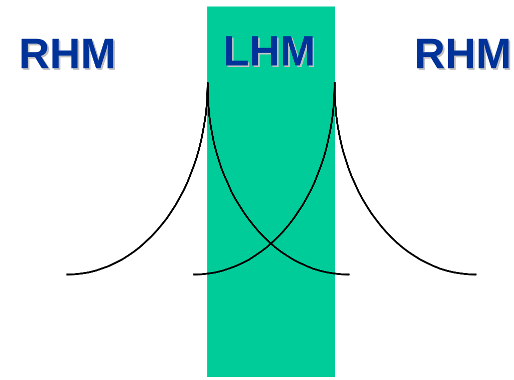
Amplification of evanescent waves





Surface polariton





Evanescent wave from near-field source

↓

Surface polariton at the 1st interface

↓

Surface polariton at the 2nd interface

The 2nd SP is stronger than the 1st SP → Amplification !

Physical model

Forced vibration and resonance of coupled oscillators

$$\ddot{\phi}_1 + \gamma \dot{\phi}_1 + \omega_0^2 \phi_1 + \Omega_c^2 \phi_2 = F e^{i\omega t},$$

$$\ddot{\phi}_2 + \gamma \dot{\phi}_2 + \omega_0^2 \phi_2 + \Omega_c^2 \phi_1 = 0,$$

$$\phi_1(\omega_0) = \frac{-i\gamma\omega_0 F e^{i\omega_0 t}}{\Omega_c^4 + \gamma^2 \omega_0^2}, \qquad \phi_2(\omega_0) = \frac{\Omega_c^2 F e^{i\omega_0 t}}{\Omega_c^4 + \gamma^2 \omega_0^2}.$$

Physical model vs. numerical results

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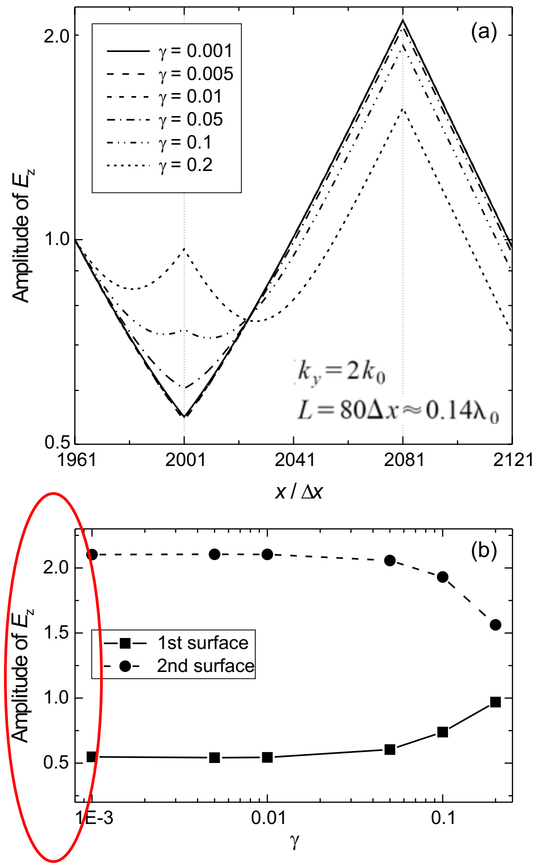
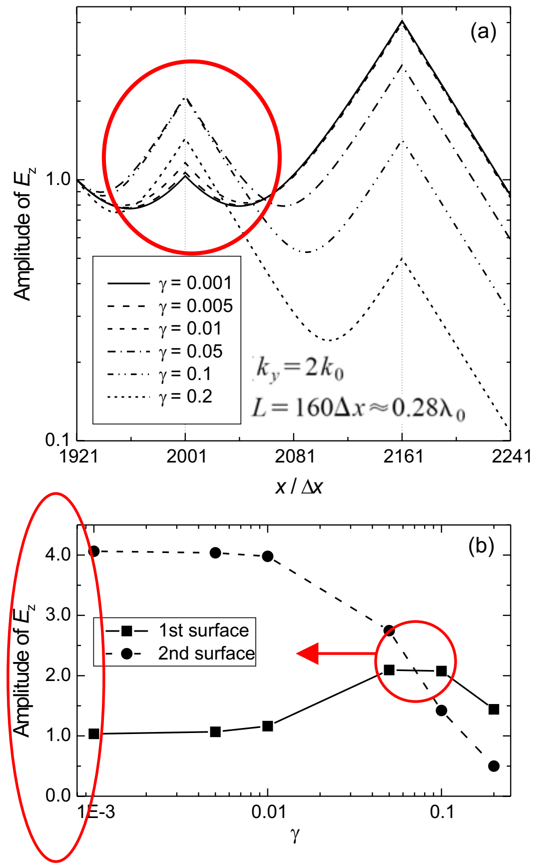




$$\phi_1(\omega_0) = \frac{-i\gamma\omega_0 F e^{i\omega_0 t}}{\Omega_c^4 + \gamma^2\omega_0^2}$$

$$\phi_2(\omega_0) = \frac{\Omega_c^2 F e^{i\omega_0 t}}{\Omega_c^4 + \gamma^2\omega_0^2}.$$

$$\frac{\partial|\phi_1(\omega_0)|}{\partial\gamma} = \frac{\omega_0 F}{(\Omega_c^4 + \gamma^2\omega_0^2)^2}(\Omega_c^4 - \omega_0^2\gamma^2)$$

$$|\phi_1/\phi_2| = \gamma\omega_0/\Omega_c^2$$

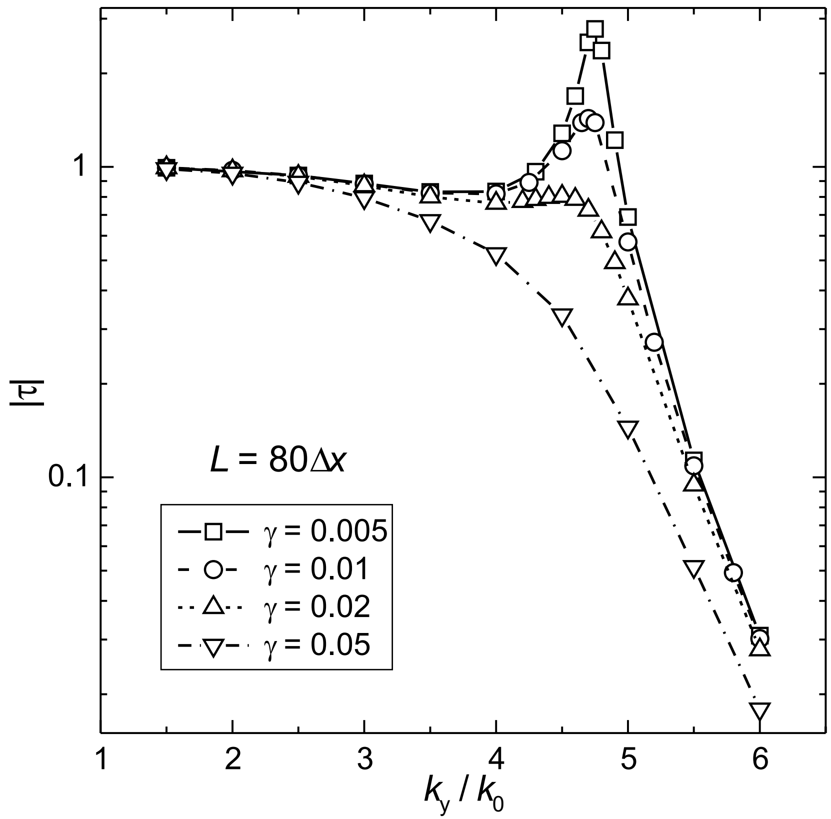
$$\Omega_c^2 = Ce^{-\kappa L}, F = De^{-\kappa L/2}$$

$$\frac{\partial|\phi_1(\omega_0)|}{\partial L} = \frac{\gamma\omega_0 F}{\Omega_c^4 + \gamma^2\omega_0^2}\frac{\kappa}{2}(3\Omega_c^4 - \omega_0^2\gamma^2)$$

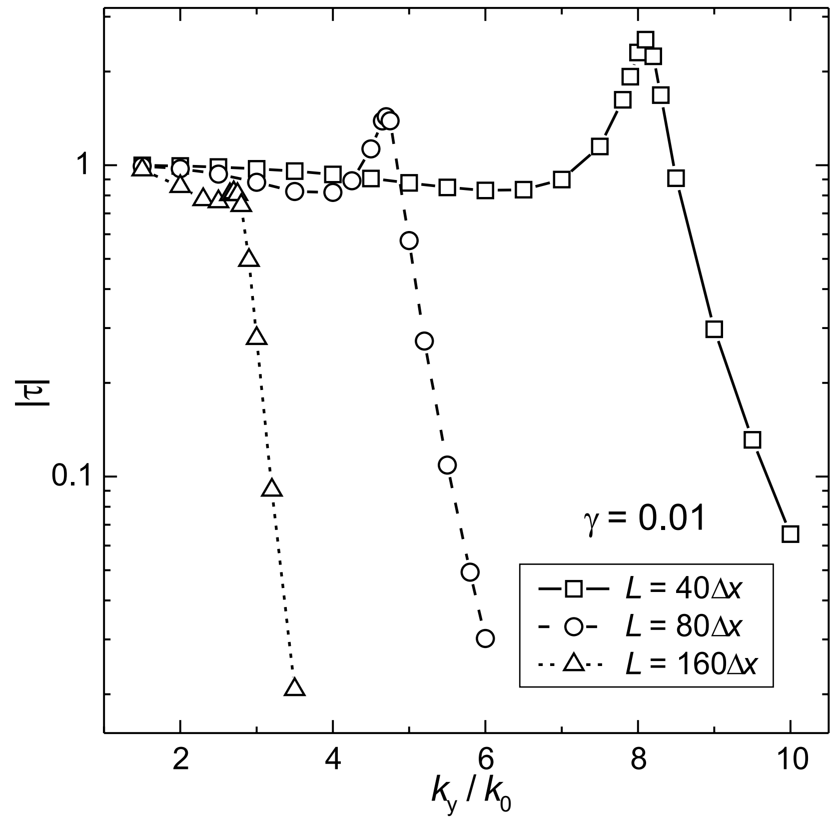
$$\frac{\partial|\phi_2(\omega_0)|}{\partial L} = \frac{\Omega_c^2 F}{(\Omega_c^4 + \gamma^2\omega_0^2)^2}\frac{\kappa}{2}(\Omega_c^4 - 3\omega_0^2\gamma^2)$$

Resolution enhancement

Dependence on γ



Dependence on *L*



Settle the discrepancies

Ziolkovski & Heyman [*PRE* **64**, 056625 (2001)]

No stable image observed (dipersive nature of LHM?)

1. **Lossless**
2. **γ ~ 0.001**

Loschialpo *et al.* [*PRE* **67**, 025602 (2003)]

stable image but no resolution enhancement observed

L ~ 3.2 λ_0

Summary

➢ Amplification of evanescent waves can be realized in LHM slab, through excitation of coupled surface polaritons.

➢ Stringent constraints apply for the amplification of evanescent waves. Only evanescent waves with limited transverse wave numbers can be amplified in lossy LHM slabs of finite width.

➢ Enhanced resolution can be achieved by a LHM superlens. The enhancement is also limited by absorption and finite width of the LHM slab.

➢ Stable image can't be obtained in the ideal lossless case, so that "perfect" lens is not realizable.

X.S. Rao and C.K. Ong, *Phys. Rev.* **B 68**, 113103 (2003); *Phys. Rev.* **E 68**, 0676xx (2003).