Sample Exam I

1. a) Take $t=0$, this gives a point $Q=(1,2,0)$ on the line. Also we have $P=(4,1,1)$ in the plane (note $Q$ is in the plane also). The direction vector of the line $\vec{v}=\langle 1,-1,4\rangle$ and $\vec{PQ}$ are both in the plano, hence

$$\vec{n}=\vec{v}\times\vec{PQ}=\langle 1,-1,4\rangle\times\langle -3,1,-1\rangle=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 4 \\ -3 & 1 & -1 \end{vmatrix}$$

$$=\begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix}\vec{i}-\begin{vmatrix} 1 & 4 \\ -3 & -1 \end{vmatrix}\vec{j}+\begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix}\vec{k}$$

$$=(1-3)\vec{i}-(-1+12)\vec{j}+(1-3)\vec{k}$$

$$=\langle -2,-11,-2\rangle$$ is a normal vecter

Take $P$ in the plane. The equation is

$$-2(x-4)-11(y-1)-2(z-1)=0$$

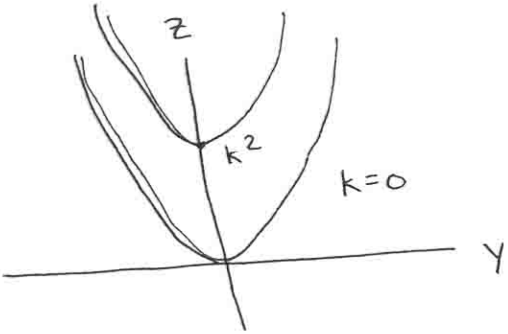
$$-2x-11y-2z+21=0$$

$$2x+11y+2z-21=0$$

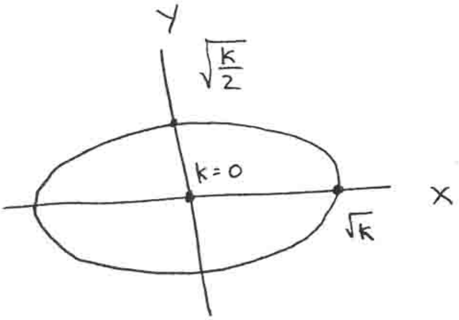
b) IGNORE!

$x^2+2y^2-z=0$

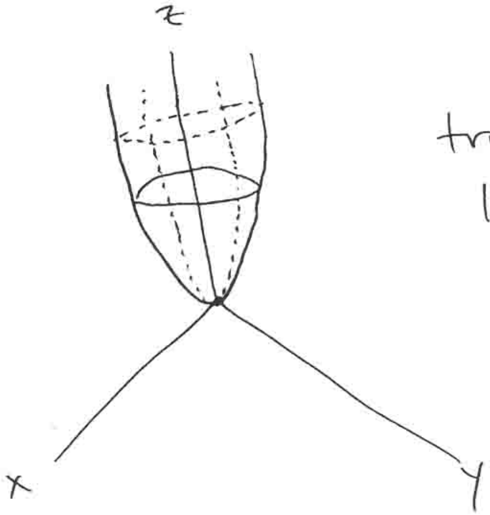
a) $x=k$ : $z=2y^2+k^2$ parabolas!




b) $z=k$ : $x^2+2y^2=k$ ellipses $k \geq 0$




) Elliptic paraboloid




traces are dotted lines!

3. a) $\vec{r}(t) = \langle \frac{1}{t}, t^2 \rangle$

$$\vec{v}(t) = \vec{r}\,'(t) = \langle -\frac{1}{t^2}, 2t \rangle$$

$$\text{Speed} = S = |\vec{v}(t)| = \sqrt{\frac{1}{t^4} + 4t^2}$$

$$\frac{dS}{dt} = \frac{1}{2}\left(\frac{1}{t^4} + 4t^2\right)^{-1/2}\left(-4\frac{1}{t^5} + 8t\right)$$

$$\left.\frac{dS}{dt}\right|_{t=1} = \frac{1}{2}(1+4)^{-1/2}(-4+8)$$

$$= \frac{2}{\sqrt{5}}$$

b) <u>IGNORE!</u>

4. a) If $z=0$, $\quad x-y=2$, $\quad 2x-y=1$ give $x-2=2x-1$

$x=-1$

$y=-3$

$(-1,-3,0)$ is on the line

If $y=0$, $\quad x+z=2$, $\quad 2x-z=1$ give $3x=3$

$x=1$

$z=1$

$(1,0,1)$ is on the line

b) $P(-1,-3,0)$ $Q(1,0,1)$ and $R(-1,0,2)$

are on the plane, so we have as normal vector:

$$\vec{n} = \vec{PQ} \times \vec{PR} = \langle 2, 3, 1 \rangle \times \langle 0, 3, 2 \rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 0 & 3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix} \vec{k}$$

$$= (6-3)\vec{i} - (2 \cdot 2 - 0)\vec{j} + (6-0)\vec{k}$$

$$= \langle 3, -4, 6 \rangle$$

and use point $Q(1,0,1)$:

$$3(x-1) - 4(y-0) + 6(z-1) = 0$$

$$3x - 4y + 6z - 9 = 0$$

5. a) We want $t$ such that

$$e^{2t} = 1$$
$$e^{-t} = 1$$
$$t^2 + 4 = 4$$

clearly $t = 0$ works

$\vec{r}(0) = \langle 1, 1, 4 \rangle$ So $(1, 1, 4)$

is on the curve

b) $\vec{r}\,'(t) = \langle 2e^{2t}, -e^{-t}, 2t \rangle$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle 2e^{2t}, -e^{-t}, 2t \rangle}{\sqrt{4e^{4t} + e^{-2t} + 4t^2}}$$

$$\vec{T}(0) = \frac{\langle 2, -1, 0 \rangle}{\sqrt{4+1}} = \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, 0 \right\rangle$$

z-axis given by $\vec{k}$

So $\vec{T}(0) \cdot \vec{k} = \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, 0 \right\rangle \cdot \langle 0, 0, 1 \rangle = 0$

So orthogonal.

Note: You don't have to use the unit tangent vecter here, do you see why?