Another Sample Exam:

1. a) Take $\vec{n} = \vec{PQ} \times \vec{PR} = \langle 2,0,-1\rangle \times \langle 0,3,-1\rangle$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -1 \\ 3 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} \vec{k}$$

$$= (0-(-3))\vec{i} - (-2)\vec{j} + (6-0)\vec{k}$$

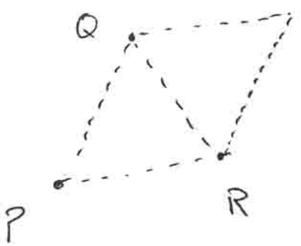
$$= \langle 3, 2, 6 \rangle$$

Use $P = (0,0,1)$

$$3(x-0) + 2(y-0) + 6(z-1) = 0$$

$$3x + 2y + 6z - 6 = 0$$

b)



Area of Δ is $\frac{1}{2}$ area of $\square$

So $\text{Area} = \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right| = \frac{1}{2} \left| \langle 3,2,6 \rangle \right| = \frac{1}{2}\sqrt{9+4+36}$

$$= \frac{7}{2}$$

2. a) direction vector $\vec{v} = \vec{n} = \langle 1, 1, 1 \rangle$

point $P(1, 2, 4)$

$x = 1 + t \quad y = 2 + t \quad z = 4 + t$

b) We want a point on the plane and the line!

$$(1+t) + (2+t) + (4+t) - 1 = 0$$

$$3t + 6 = 0$$

$$t = -2$$

$$R = (-1, 0, 2)$$

Then Distance $D = |\vec{PR}| = |\langle -2, -2, -2 \rangle|$

$$= \sqrt{4+4+4}$$

$$= \sqrt{12} = 2\sqrt{3}$$

3. a) $\vec{a}(t) = \langle 3\cos t, -3\sin t, 0 \rangle$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 3\sin t, 3\cos t, 0 \rangle + \vec{c}$$

$$\vec{v}(0) = \langle 0, 3, 0 \rangle + \vec{c} = \langle 0, 0, 2 \rangle$$

$$\vec{c} = \langle 0, -3, 2 \rangle$$

$$\vec{v}(t) = \langle 3\sin t, 3\cos t - 3, 2 \rangle$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle -3\cos t, 3\sin t - 3t, 2t \rangle +$$

$$\vec{r}(0) = \langle -3, 0, 0 \rangle + \vec{D} = \langle 2, 0, 1 \rangle$$

$$\vec{D} = \langle 5, 0, 1 \rangle$$

$$\vec{r}(t) = \langle -3\cos t + 5, 3\sin t - 3t, 2t + 1 \rangle$$

b) $\cancel{\text{Distance} = \int_0^{2\pi} |\vec{r}'(t)|\,dt = \int_0^{2\pi} |\vec{v}(t)|\,dt}$

$$\cancel{= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t - 18\cos t + 9 + 4}\,dt}$$

$$\cancel{= \int_0^{2\pi} \sqrt{22 - 18\cos t}}$$

$$\cancel{= \int_0^{2\pi}}$$

b) Distance $= \int_0^{2\pi} |\vec{v}(t)|\,dt$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t - 18\cos t + 4}\; dt$$

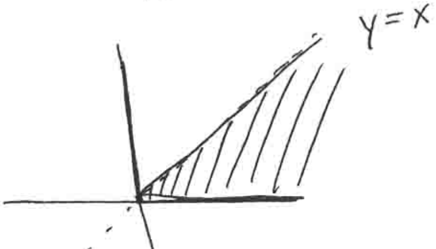
$$= \int_0^{2\pi} \sqrt{22 - 18\cos t}\; dt$$

You do not need to know how to evaluate this integral, there is an error in the problem. It is supposed to come out to be something easy.

4. a) We need $x \geq 0$, $y \geq 0$, and $x - y \geq 0$

or $x \geq y$

So its all (x,y) where these conditions hold



b) Approach along $x = 0$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{x^2+xy+y^2}{x^2-y^2} = \lim_{y\to 0} \frac{y^2}{-y^2} = -1 = L_1$$

Approach along $y=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x^2+xy+y^2}{x^2-y^2} = \lim_{x\to 0} \frac{x^2}{x^2} = 1 = L_2$$

limit DNE Since $L_1 \neq L_2$

5. a) We need t such that

$$t^2+t-3=-1 \qquad \frac{1}{t}=1 \qquad \sqrt{t+3}=2$$

$t=1$ clearly satisfies all three equations.
So $(-1,1,2)$ is on the curve.

b)
$$\vec{v}(t) = \vec{r}\,'(t) = \left\langle 2t+1,\ -\frac{1}{t^2},\ \frac{1}{2}(t+3)^{-1/2} \right\rangle$$

$$\text{Speed} = |\vec{v}(t)| = \sqrt{(2t+1)^2 + \frac{1}{t^4} + \frac{1}{4}\cdot\frac{1}{t+3}}$$

$$\text{Speed}\Big|_{t=1} = \sqrt{3^2+1+\frac{1}{4}\cdot\frac{1}{4}} = \sqrt{10+\frac{1}{16}}$$

$$= \frac{\sqrt{161}}{4}$$

c) $\vec{T}(t) = \dfrac{\vec{v}(t)}{|\vec{v}(t)|}$

$$\vec{T}(1) = \frac{\langle 3, -1, \frac{1}{4} \rangle}{\frac{\sqrt{161}}{4}} = \left\langle \frac{12}{\sqrt{161}}, -\frac{4}{\sqrt{161}}, \frac{1}{\sqrt{161}} \right\rangle$$