can be seen as part of E in Fig. 1, so it will be shaped by the digital noise canceller for first order, as shown in (1), and will thereby be effectively suppressed, too.

V. Conclusion

The analysis of the sensitivity of TIM to coefficient mismatches has been reviewed. A new cascade-parallel architecture of sigma–delta ADC's is proposed on the basis of TIM. Though the derivation procedure of it is based on an example of (1-1) cascade and M is assumed to be four, the cascade-parallel architecture can be a general method to overcome the shortcoming of TIM through extensions. Simulation results of the examples indicated that the new architecture effectively suppresses the influence of the noise introduced by coefficient mismatches, while retaining the speed advantages of TIM. In addition, it turned out to be quite simple. Thus, it can be a good approach to implement TIM in practical applications.

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Time-Scaled Electrical Networks—Properties and Applications in the Design of Programmable Analog Filters

Shanthi Pavan and Yannis Tsividis

***Abstract*—In this paper, we discuss the properties of time-scaled electrical networks. Two specific ways of implementing scaled linear networks, constant-conductance scaling and constant-capacitance scaling, are reviewed along with their noise properties. We then extend time scaling to nonlinear networks. We show that constant-capacitance scaled networks are optimal with respect to noise and dynamic range, irrespective of the scaling factor.**

***Index Terms*—Continuous time, filter, nonlinear networks, programmable.**

I. Introduction

Continuous-time integrated filters need to be programmable over wide frequency ranges for several applications, e.g., in the read channel of hard disk drives. Such filters must maintain the relative shape of the frequency response indentical, irrespective of the set bandwidth, while maintaining adequate dynamic range. Approaches to the design of programmable filters are considered in this brief.

A. Time Scaling: Definition

We will discuss both linear and nonlinear networks. Since, in the latter, the term "frequency response" does not have meaning in the strict sense of the term, we will use the term "time scaling" rather than "frequency scaling" throughout this paper.

Consider an initially relaxed network $\mathcal{N}$. For the time being, we consider a single input, single output network. Let an arbitrary input voltage $v_i(t)$ produce an output voltage $v_o(t)$. Further, let us assume the existence of a network $\hat{\mathcal{N}}$ satisfying the following condition: Any input $\hat{v}_i(t) = v_i(\alpha t)$ results in an output $\hat{v}_o(t) = v_o(\alpha t)$. The only restriction on α is that it be positive. Then, we call $\hat{\mathcal{N}}$ the time-scaled version of $\mathcal{N}$, with a scaling factor α. This is denoted in Fig. 1. When $\mathcal{N}$ and $\hat{\mathcal{N}}$ are linear, scaling in time by α corresponds to scaling of the frequency response of the network by $1/\alpha$. *The relative shape of the magnitude response remains the same*.

Throughout this paper, we discuss only (trans)conductance-capacitance networks, due to their significance in practical filter designs (for example, the disk-drive-read channel application). Extension to active-*RC* networks is easy and is not considered in this work. Two methods for realizing time (frequency-response) scaled networks are (Fig. 2) [1] the following.

1) Constant-capacitance scaling: multiply *all* conductances and transconductances by α, while keeping all capacitors unchanged.
2) Constant-conductance scaling: multiply *all* capacitors by $1/\alpha$, while keeping all conductances and transconductances unchanged.

The above two scaling techniques are not the only possibilities. For instance, one could easily think of a strategy where all conductances are

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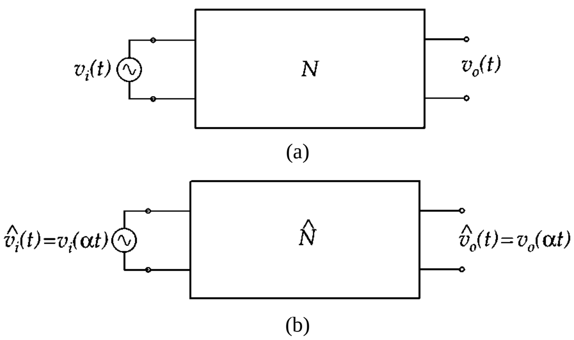

Fig. 1. Definition of time scaling. (a) Original network. (b) Time-scaled network.

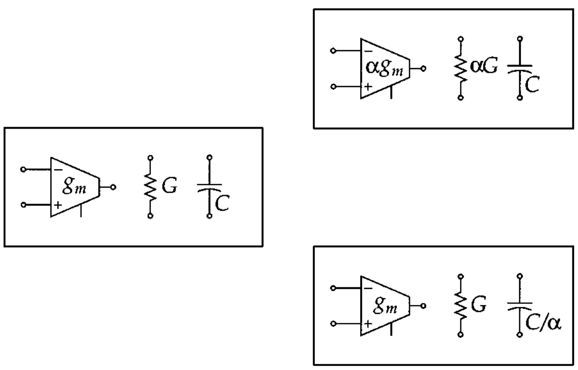

Fig. 2. Illustration of constant-capacitance and constant-conductance scaling for a general linear (trans)conductance-capacitance network. Left: Original network. Right top: Constant-capacitance scaled network. Right bottom: Constant-conductance scaled network.

scaled by $\sqrt{\alpha}$ while all capacitors are scaled by $\sqrt{1/\alpha}$. Only constant-conductance and constant-capacitance scaling are dicussed in this work in view of their practical significance.

In Section II, we review the noise properties of scaled networks. In Section III, we extend the concept of time scaling to (trans)conductance-capacitance networks that are nonlinear. Section IV considers the distortion properties of weakly nonlinear time-scaled networks. In Section V, we prove that constant-capacitance scaling results in an optimal implementation of programmable continuous-time filters. Section VI contains the conclusions of this work.

II. Noise Properties of Scaled Networks (A Review)

In this section, we review the noise properties of constant-conductance and constant-capacitance scaled networks [2], [3]. Similar relations have been derived for active-*RC* networks [4]. Here, we give the formulae for (trans)conductance-capacitance networks. This will facilitate the proof of optimality of constant-capacitance scaling in Section IV. Only white noise is considered in this paper. Let us assume that the noise of every transconductance and conductance can be represented by the models shown in Fig. 3. k is the Boltzmann's constant and T is the absolute temperature. η is the excess noise factor of the transconductor. Let $H(f)$, $S_{\rm no}(f)$ and $\overline{v_{\rm no}^2}$ be the original network's frequency response, output noise power spectral density, and output noise power, respectively.

Consider Fig. 4(a), where we isolate a single noise source v_{n1} from the network. We are interested in finding its contribution to the output noise spectral density $S_{\rm no}(f)$ and the integrated output noise voltage $\overline{v_{\rm no}^2}$. We denote the transfer function from v_{n1} to v_o as $H_{1o}(f)$. We can write

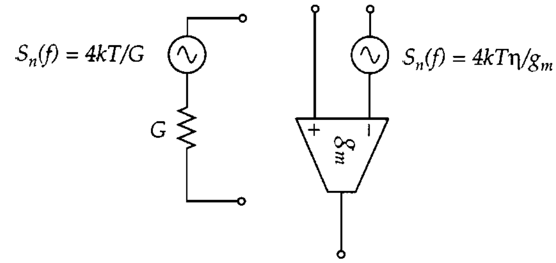

Fig. 3. Noise model for the conductances and transconductors.

$$S_{\rm no}(f) = |H_{1o}(f)|^2 \cdot \frac{4kT}{G_1} \tag{1}$$

$$\begin{aligned}\overline{v_{\rm no}^2} &= \int_0^\infty S_{\rm no}(f)\,df \\ &= \int_0^\infty |H_{1o}(f)|^2 \cdot \frac{4kT}{G_1} df \\ &= \frac{4kT}{G_1}\int_0^\infty |H_{1o}(f)|^2\,df.\end{aligned} \tag{2}$$

For the constant-capacitance scaled network [Fig. 4(b)], the relations corresponding to (1) and (2) are

$$\hat{S}_{\rm no}(f) = |H_{1o}(f/\alpha)|^2 \cdot \frac{4kT}{\alpha G_1} \tag{3}$$

$$\begin{aligned}\overline{\hat{v}_{\rm no}^2} &= \int_0^\infty \hat{S}_{\rm no}(f)\,df \\ &= \int_0^\infty |H_{1o}(f/\alpha)|^2 \cdot \frac{4kT}{\alpha G_1} df \\ &= \frac{4kT}{G_1}\int_0^\infty |H_{1o}(f/\alpha)|^2\,d(f/\alpha).\end{aligned} \tag{4}$$

From (1) and (3), we get

$$\hat{S}_{\rm no}(f) = \frac{1}{\alpha} S_{\rm no}(f/\alpha). \tag{5}$$

Comparing (4) and (2), we see that

$$\overline{v_{\rm no}^2} = \overline{\hat{v}_{\rm no}^2}. \tag{6}$$

Following the same reasoning as above and utilizing power spectral density superposition, it is easy to extend the results to multiple independent noise sources and conclude that *the integrated output noise power of a constant-capacitance scaled network is independent of the scaling factor* α.

Relations for the integrated output noise and noise spectral density for constant-conductance scaled networks can be derived in an analogous manner. It can be shown that the integrated output noise power of a constant-conductance scaled network is directly proportional to the scaling factor α. The noise properties of constant-capacitance and constant-conductance scaled networks are summarized in Table I.

As stated, a filter designed using constant-capacitance scaling principles will have the same root mean square (RMS) output noise irrespective of its bandwidth, whereas in the case of constant-conductance scaling, the total noise power varies with the scaling factor α. We now consider a programmable filter, with α settable between the values of 1 and $\alpha_{\max}$, where $\alpha_{\max}$ » 1. If the design of this filter is based on

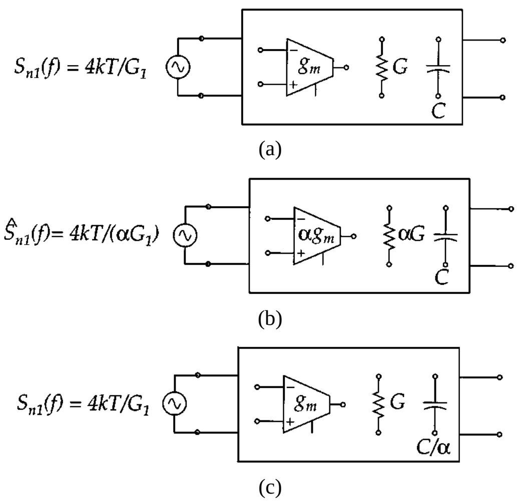

Fig. 4. Noise properties of constant-capacitance scaled networks. (a) Original network. (b) Constant-capacitance scaled network. (c) Constant-conductance scaled network.

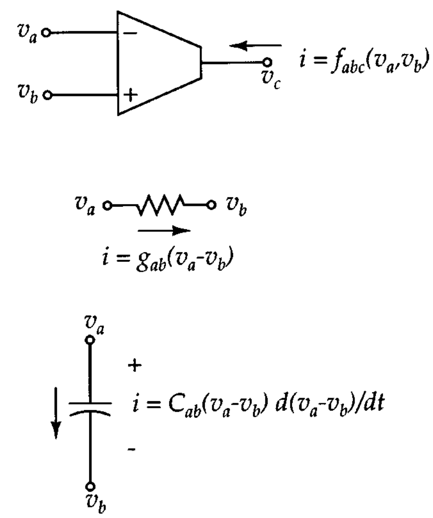

Fig. 5. Model for nonlinear (trans)conductors and capacitors.

TABLE I
NOISE PROPERTIES OF SCALED NETWORKS

Parameter	Original Network	Constant Capacitance Scaled Network	Constant Conductance Scaled Network
Frequency Response	$H(f)$	$H(f/\alpha)$	$H(f/\alpha)$
Output Noise Spectral Density	$S_{no}(f)$	$\frac{1}{\alpha}S_{no}(f/\alpha)$	$S_{no}(f/\alpha)$
Output Noise Power	$\overline{v_{no}^2}$	$\overline{v_{no}^2}$	$\alpha\overline{v_{no}^2}$

constant-conductance scaling, it must be such that the filter meets the noise specification at the setting $\alpha = \alpha_{\max}$. As a result, the filter will be grossly overdesigned at lower settings of α.

In contrast to this, if the programmable filter is designed using constant-capacitance scaling, the output noise will be independent of the scaling factor (Table I). Thus, if the filter is designed optimally for one setting of α, it will remain optimal for all other settings, and no overdesign will be needed.

It has been shown elsewhere [2] that the output noise power is inversely proportional to the total filter capacitance. Combining this fact with the above observations, it is easy to see that a constant-conductance programmable filter will need $\alpha_{\max}$ times more capacitance than a constant-capacitance design. This is why programmable filters should be designed using constant-capacitance scaling. It can also be shown that, although the two filters will have the same total power dissipation at the most wideband setting, in the constant-capacitance case, power dissipation will decrease as α is decreased, whereas in the constant-conductance case it will remain constant, independent of α.

III. Time Scaling in Nonlinear Networks

The concepts of frequency response and transfer function do not exist in the common sense for nonlinear systems; hence, we can only talk about "time scaling." Consider the case when the (trans)conductors and capacitors are nonlinear. The input–output relations for nonlinear transconductors, conductors, and capacitors are written as shown in Fig. 5. Consider now a general network consisting of these elements shown in Fig. 6(a). We can use the modified nodal analysis (MNA) formulation for writing the network equations [5].

A typical equation for the kth node of the network will be of the form

$$\sum_p C_{kp}(v_k(t) - v_p(t))\frac{d(v_k(t) - v_p(t))}{dt} + \sum_{q,r} f_{qrk}(v_q(t), v_r(t)) + \sum_u g_{ku}(v_k(t) - v_u(t)) = 0, \quad k\text{th node} \tag{7}$$

and assuming that the input $v_i(t)$ is connected to node 1

$$v_1(t) = v_i(t). \tag{8}$$

Let us replace t in these equations by αt. We obtain

$$\sum_p C_{kp}(v_k(\alpha t) - v_p(\alpha t))\frac{d(v_k(\alpha t) - v_p(\alpha t))}{d\alpha t} + \sum_{q,r} f_{qrk}(v_q(\alpha t), v_r(\alpha t)) + \sum_u g_{ku}(v_k(\alpha t) - v_u(\alpha t)) = 0, \quad k\text{th node} \tag{9}$$

and

$$v_1(\alpha t) = v_i(\alpha t). \tag{10}$$

Equations (9) and (10) can be rewritten as

$$\sum_p \frac{C_{kp}(v_k(\alpha t) - v_p(\alpha t))}{\alpha}\frac{d(v_k(\alpha t) - v_p(\alpha t))}{dt} + \sum_{q,r} f_{qrk}(v_q(\alpha t), v_r(\alpha t)) + \sum_u g_{ku}(v_k(\alpha t) - v_u(\alpha t)) = 0, \quad k\text{th node} \tag{11}$$

$$v_1(\alpha t) = v_i(\alpha t). \tag{12}$$

Notice that these are the equations one would get for the kth node of the network of Fig. 6(c), where all the (trans)conductances are the same as in the original network, while all the capacitors have been scaled by the factor $1/\alpha$ and when the network is excited by an input $\hat{v}_i(t) = v_i(\alpha t)$. We refer to this as the "constant-conductance" scaled network. Both these networks can be solved using the MNA formulation. Since all voltages in the scaled network are time-scaled by a factor α, we obtain

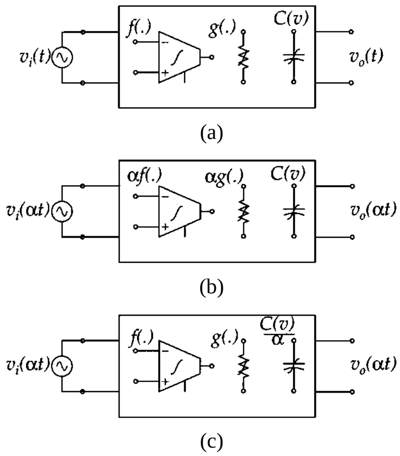

Fig. 6. Constant-capacitance and constant-conductance scaling in nonlinear networks. (a) Original network. (b) Constant-capacitance scaled network. (c) Constant-conductance scaled network.

the following, as in the linear case: if an input $v_i(t)$ to the original network produces an output $v_o(t)$, an input $\hat{v}_i(t) = v_i(\alpha t)$ applied to the scaled network produces an output $\hat{v}_o(t) = v_o(\alpha t)$. Note that the scaled network must have the same set of initial conditions as the unscaled network for the above to hold.

Alternatively, (9) can be rewritten as

$$\sum_p C_{kp}(v_k(\alpha t) - v_p(\alpha t))\frac{d(v_k(\alpha t) - v_p(\alpha t))}{dt} + \sum_{q,r} \alpha f_{qrk}(v_q(\alpha t), v_r(\alpha t)) + \sum_u \alpha g_{ku}(v_k(\alpha t) - v_u(\alpha t)) = 0, \quad k\text{th node} \tag{13}$$

$$\hat{v}_1(\alpha t) = v_i(\alpha t). \tag{14}$$

These equations are those that one would obtain for the network of Fig. 6(b), where all the capacitors are the same as in the original network, while all conductors and transconductors have been scaled by the factor α. We refer to this as the "constant-capacitance" scaled network. As in the constant-conductance case, an input $v_i(\alpha t)$ to the scaled network results in an output $v_o(\alpha t)$.

IV. Distortion in Weakly Nonlinear Constant-Capacitance Scaled Filters

Fig. 7 shows two weakly nonlinear filters, with (b) the constant-capacitance scaled version of (a). The network of Fig. 7(a) is excited by a sinewave of amplitude A and frequency ω_o. We express the output as a Fourier series

$$v_o(t) = \sum_n A_n \sin(n\omega_o t) + B_n \cos(n\omega_o t). \tag{15}$$

Since the filter of Fig. 7(b) is a scaled version with a scaling factor α, its output for an excitation $A\sin(\alpha\omega_o t)$ is

$$\hat{v}_o(t) = v_o(\alpha t) = \sum_n A_n \sin(n\alpha\omega_o t) + B_n \cos(n\alpha\omega_o t). \tag{16}$$

Observe that the Fourier coefficients of the output remain the same for both networks. This means that if the original filter has a total harmonic distortion (THD) of x% when excited by a frequency ω_o, the scaled filter will also have a THD of x% when excited by a tone of the same amplitude, but a frequency of $\alpha\omega_o$. Thus, if the original filter has a worst-case THD of x% for an input tone of frequency ω_o and amplitude A, then the scaled filter will also have a worst-case THD of x% when excited by a sinewave of amplitude A, but a frequency $\alpha\omega_o$. Therefore, *the worst-case distortion of a scaled filter is independent of the scaling factor*.

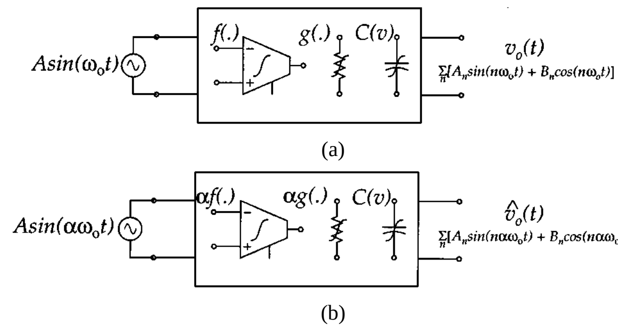

Fig. 7. Distortion in weakly nonlinear filters. (a) Original filter. (b) Scaled filter.

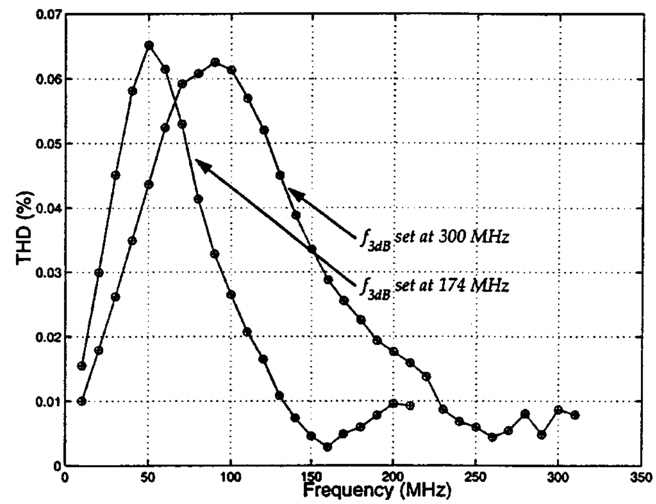

Fig. 8. Distortion simulations of a constant-capacitance scaled fourth-order Butterworth filter.

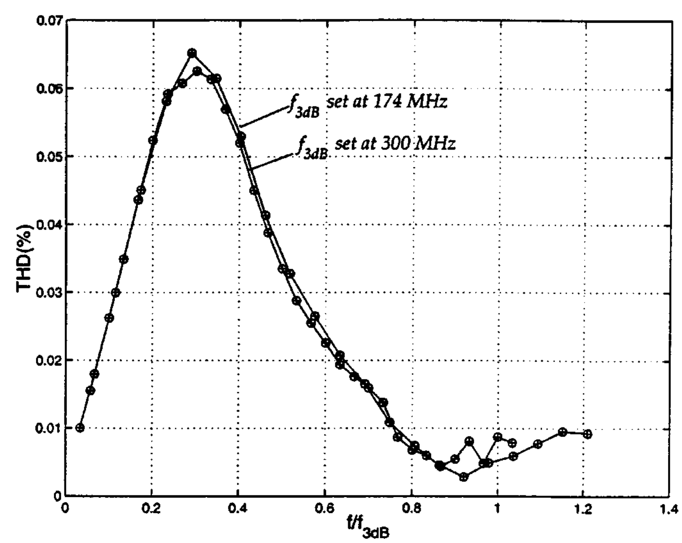

Fig. 9. The distortion curves of Fig. 8, replotted with the x axis normalized to bandwidth.

For a more general periodic input, if $v_o(t)$ is represented as a sum of components $\sum_n k_n\phi_n(t)$, where the ϕ_n are *not necessarily harmonically related* (they could be intermodulation components, cross-modulation components, or anything else), then $\hat{v}_o(t) = v_o(\alpha t) = \sum_n k_n\phi_n(\alpha t)$ for $\hat{v}_i(t) = v_i(\alpha t)$. Therefore, the amplitudes of the inter/cross modulation components (the k_n) remain the same for the scaled network when the input signal is time scaled.

To verify our conclusions on the harmonic distortion properties of scaled nonlinear networks, simulations were run on a device-level CMOS implementation of a constant-capacitance scaled fourth-order Butterworth low-pass filter, whose bandwidth was programmable from 60 to 350 MHz [6]. The filter bandwidth was set to 300 MHz, and the total harmonic distortion in the output waveform was plotted as a function of the input frequency. Then the bandwidth setting was changed to 174 MHz and the above procedure was repeated again. The results are shown in Fig. 8. In Fig. 9, the x axis is normalized with respect to the filter bandwidth setting for both the curves shown in Fig. 8. As predicted by the theory, the two normalized distortion curves are in very good agreement. These are device-level simulations. The absolute distortion levels are not important because they depend on the accuracy of the transistor models used. What is of great significance is that the distortion curves are nearly identical when normalized to the bandwidth. To obtain accurate results, the simulator time-step was forcibly set in inverse proportion to the bandwidth. This ensures that the simulator converges to roughly the same degree at the end of a time step regardless of the filter bandwidth.

V. Dynamic Range in Constant-Capacitance Scaled Filters

Along with the noise properties of scaled networks discussed earlier in this brief, the observation on distortion has important consequences in filter design. If the input signal to a filter is very small, the output is masked by the internal noise of the filter. If the input signal is very large, distortion effects set in. The dynamic range of the filter is defined as the ratio of the maximum output signal level permissible (with some acceptable distortion level) to the minimum output signal (usually the root mean squared output noise). If a constant-capacitance scaled network is used to implement a programmable filter, we have shown that the integrated output noise power is constant, irrespective of the set bandwidth. From our discussion above of time scaling in nonlinear circuits, we concluded that the maximum output signal level for a given level of distortion is independent of the set bandwidth too. From the above two statements, we conclude that the dynamic range of a constant-capacitance scaled network is independent of the scaling factor. Thus, a *constant-capacitance scaled* filter represents a very desirable situation, summarized below.

1) The output noise power is independent of the scaling factor.
2) The worst-case distortion (and hence, the maximal signal level) is independent of the scaling factor.
3) The dynamic range is independent of the scaling factor.

Notice that while there are many scaling strategies to maintain frequency response, *constant-capacitance* scaling keeps the dynamic range constant, irrespective of the bandwidth. Hence, no overdesign is needed. If the original filter is designed optimally in terms of white noise and distortion, then any constant-capacitance scaled version of it will also be optimal, irrespective of its frequency setting. In addition, constant-capacitance scaling lends itself to a design technique that is remarkably immune to effects of parasitic capacitances; the reader is referred elsewhere [6] where a filter chip, programmable from 60 to 350 MHz, is presented.

VI. Conclusion

We have discussed some properties of time-scaled electrical networks. We have shown analytically that constant-capacitance scaled filters have noise, distortion, and dynamic range independent of the scaling factor (not counting $1/f$ noise sources). These observations have been verified through simulation and experiments reported elsewhere [6].

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