AN ANALYTICAL SOLUTION FOR A CLASS OF OSCILLATORS, AND ITS APPLICATION TO FILTER TUNING

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ABSTRACT

We present a completely analytical solution to a filter–comparator oscillator system and verify it by macro-model simulations and experiment. We discuss the applications of this kind of oscillator in a vector–locked loop system for continuous time filter tuning.

1. INTRODUCTION

In a sine-wave oscillator, positive feedback is used around a frequency selective circuit to drive the poles of the corresponding closed loop linear system into the right half s-plane. In the case to be considered in this paper, the "gain" of the amplifier is set to ∞, as shown in Figure 1, leading to the filter–comparator oscillator. Such systems are encountered in nonlinear control systems literature [1] [2] [3] and have been used by designers [4] in filter tuning schemes.

The filter-comparator system could be analyzed by using the describing function approach [2], where the nonlinear block is replaced by an "equivalent" linear block. A first–order describing function analysis, however, predicts that the system will oscillate at the filter pole frequency, regardless of the filter quality factor, which is incorrect. A higher order describing function analysis gets close to the exact result. An exact method for systems consisting of linear networks and relays has been proposed by Tsypkin [3] and is described in detail in [2]. This method is very complicated for this particular system.

The solution we present in this paper is straightforward, provides intuition, and gives information about all the quantities of interest *exactly*.

2. OSCILLATOR TRANSIENT AND STEADY STATE

The system we analyze is shown in Figure 1. For simplicity, until further notice we assume that the comparator output levels are 0 and 1. The filter is of the second–order bandpass type. Its transfer function is

$$H(s) = \frac{\frac{s}{\omega_0}}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1} \qquad (1)$$

In the technique to be proposed below, we will employ the step response of the filter, $s(t)$, which is

$$s(t) = \frac{1}{\sqrt{1-\frac{1}{4Q^2}}} \exp\left(\frac{-\omega_0 t}{2Q}\right) \sin\left(\omega_0\sqrt{1-\frac{1}{4Q^2}}\, t\right) u(t) \qquad (2)$$

where *u(t)* is the unit step function. The step response crosses zero at times

$$t_n = nt_1, \qquad n = 0, 1, 2... \qquad (3)$$

where

$$t_1 = \frac{\pi}{\omega_0\sqrt{1-\frac{1}{4Q^2}}} \qquad (4)$$

The mechanism of oscillation build up will be described with the aid of Figure 2. Let us assume that the system is initially relaxed, and that oscillation is triggered by a small positive noise at the comparator input at time $t = t_o = 0$. This will cause a step input $u(t)$ to the bandpass filter. The output $y(t)$ of the bandpass filter for $0 < t < t_1$ will coincide with the filter step response $s(t)$, as is shown in Figure 2. This waveform crosses zero at $t = t_1$, so at that instant the comparator switches again. Between this switching instant and the next one, the comparator output can be represented by the superposition of two steps, the first at t_0 and the second at t_1:

$$x(t) = u(t) - u(t - t_1) \qquad (5)$$

Thus, for the same interval, the output of the linear filter can be obtained using superposition as

$$y(t) = s(t) - s(t - t_1) \qquad (6)$$

Notice that the zero crossings of $s(t-t_1)$ are t_1 apart from each other, just as was the case with $s(t)$. Also, the time at which $s(t-t_1)$ starts coincides with the zero crossing t_1 of $s(t)$. Thus, the output $y(t) = s(t) - s(t - t_1)$ will reach its next zero crossing when *both* $s(t)$ and $s(t - t_1)$ cross zero, i.e at $t = 2t_1$. At this point, the comparator switches again, and so until the next zero crossing, its input will be

$$x(t) = u(t) - u(t - t_1) + u(t - t_2) \qquad (7)$$

and its output will be

$$y(t) = s(t) - s(t - t_1) + s(t - 2t_1) \qquad (8)$$

Reasoning as above, we conclude that the next zero–crossing will occur at $t = 3t_1$, and so on. It now becomes obvious that the output of the comparator can be represented for all positive time by

$$x(t) = \sum_{n=0}^{\infty}(-1)^n u(t - nt_1) \quad , \quad t > 0 \qquad (9)$$

The filter output then is:

$$y(t) = \sum_{n=0}^{[t/t_1]}(-1)^n s(t - nt_1) \quad , \quad t > 0 \qquad (10)$$

where $[t/t_1]$ denotes the integer part of t/t_1. It is apparent from Figure 2 that the terms in the sum that produces $y(t)$ are positive for $nt_1 < t < (n+1)t_1$ if n is even, and negative if n is odd. By writing

$$\boxed{T \equiv 2t_1} \qquad (11)$$

we see that the terms in the sum are all positive for $mT < t < mT + \frac{T}{2}$, and negative for $mT + \frac{T}{2} < t < (m+1)T$, where m is an integer. This is shown in Figure 3.

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2.1. Steady State Response

The steady state response can be obtained by using (2) in (10) and allowing t to increase. The result of this process is :

$$y_{ss}(mT+\tau) = A\exp\left(-\frac{\omega_0\tau}{2Q}\right)\sin(\omega_{osc}\tau), 0 < \tau < \frac{T}{2} \quad (12)$$
$$= A\exp\left(-\frac{\omega_0(\tau-\frac{T}{2})}{2Q}\right)\sin(\omega_{osc}\tau), \frac{T}{2} < \tau < T$$

where :

$$\boxed{\omega_{osc} = \omega_0\sqrt{1-\frac{1}{4Q^2}}} \quad (13)$$

$$\boxed{T = \frac{2\pi}{\omega_{osc}}} \quad (14)$$

$$\boxed{A = \frac{1}{\left[1-\exp\left(-\frac{\pi}{\sqrt{4Q^2-1}}\right)\right]\sqrt{1-\frac{1}{4Q^2}}}} \quad (15)$$

and m is any integer. The peak of the oscillatory waveform is obtained by finding the maximum of $y_{ss}(mT+\tau)$ in the time interval $0 < \tau < \frac{T}{2}$, and is an exercise in calculus. We will find this peak for the special case when the filter quality factor is high, using (12) and (15):

$$A \cong \frac{1}{1-\exp(-\frac{\pi}{\sqrt{4Q^2-1}})} \cong \frac{1}{1-(1-\frac{\pi}{\sqrt{4Q^2-1}})}, Q >> 1 \quad (16)$$

Thus,

$$y_{ss,peak} \cong A \cong \frac{2Q}{\pi} \quad , \quad Q >> 1 \quad (17)$$

Note that, all along, we have assumed the difference in the clipping levels of the comparator to be unity. In the more general case, the output will be directly proportional to the difference in clipping levels of the comparator. In Figure 4, we show the predicted and observed waveforms of the comparator and the filter when Q is 1. The measured and predicted amplitude of the output is shown in Figure 5. Note that as the quality factor increases, the amplitude increases as predicted by (17). The frequency as a function of filter quality factor is shown in Figure 6.

3. APPLICATIONS TO FILTER TUNING

A long standing problem in filter design has been to tune a filter to a desired response in the face of variations in temperature and other environmental factors, tolerances and aging. The tuning strategy can be indirect [5] or direct [6]. In either case, the filter to be tuned is a voltage (or current) controlled filter, that is, a filter whose parameters are "programmable" by a set of control voltages(or currents). For a second-order section the parameters of greatest interest are the pole frequency and the pole quality factor. Hence these two parameters need to be tuned.

The general block diagram of a Vector Lock Loop (VLL) based on a Voltage Controlled Filter (VCF) is shown in Figure 7 [8] [7] [9]. This scheme is chosen in order to appropriately introduce our proposed scheme in the sequel. The variable of interest in the frequency control loop is ϕ, the phase difference between the reference and the output, while in the Q–lock loop, it is M, the ratio of the output and input magnitude. We will now point out the problem with inter-loop coupling in a conventional VLL which uses a second order filter. For this argument, the reader is referred to Figure 8(a). This shows the situation with the conventional vector locked loop. Assume that, to begin with, the relative shape of the response is very close to the ideal, while the center frequency deviates significantly from the desired value. For purposes of argument, assume that frequency and Q tuning is done sequentially. The magnitude detector will have an output which is very low, and this would cause the Q-loop to increase the filter Q, although there is only a frequency error in the system. Now however, when the frequency loop converges to the desired value, the quality factor will be in error, and the magnitude loop now needs more time to get back to the right value. Notice that if the desired quality factor is large, then even a small error in pole frequency could result in the magnitude detector sensing a very low output. Thus the problems with locking tend to get compounded with increasing filter selectivity. In traditional schemes, these problems are taken care of by making the Q-loop much slower than the frequency loop, so as to make the loops quasi-independent. Note that, ideally, we would want

$$\frac{\partial\phi(\omega_0, Q)}{\partial Q} = 0 \quad (18)$$

$$\frac{\partial M(\omega_0, Q)}{\partial\omega_0} = 0 \quad (19)$$

From Figure 8(a), it is obvious that all the problems with the conventional design could be avoided if we were somehow able to "move" the reference around, so that we can always sense the peak gain of the filter, no matter at what frequency it occurs. This situation is illustrated in Figure 8(b). Now, the magnitude detector output is constant regardless of filter center frequency, and a function of quality factor only. To generate a "reference frequency" which is always equal to the filter pole frequency, one can make the filter oscillate and pass its output through a limiter to obtain a constant amplitude. This is precisely what the system of Figure 1 does. From (17) it is apparent that the amplitude of oscillation is now a single valued function of filter quality factor only, and is completely independent of pole frequency.

The entire vector lock loop is shown in Figure 9. The pole frequency of the filter is set by locking the oscillation frequency to the reference using a phase–lock loop. The quality factor is set by measuring the output magnitude. A scheme for Q tuning which uses a VGA plus a postprocessor instead of a comparator, has been proposed elsewhere [11]. We now summarize the advantages of the VLL just presented.

(a) The pole frequency can be tuned with absolutely no error in spite of offsets in the frequency control loop because the system utilizes the PLL principle, in which phase errors do not result frequency errors.

(b) The reference can be a square wave, unlike in the VCF case, which demands a reference signal with low harmonic content.

(c) The filter operates in a linear fashion, and the oscillation frequency of the entire system tracks the pole frequency of the filter with variations in ambient conditions and other environmental factors.

(d) The amplitude and frequency loops are independent.

Thus, this loop is a marriage of the VCF and the PLL schemes, combining the advantages of both in the same method, and getting rid of the disadvantages of either methods. The loop has the same circuit complexity as any other VLL scheme. A low-frequency version of the proposed Vector Locked Loop was bread-boarded. The

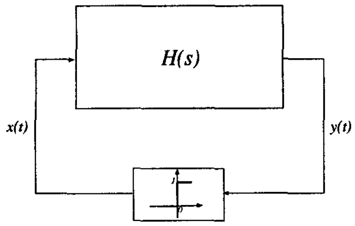

Figure 1. Block Diagram of the Oscillator

Master–Slave system was realized by using MOS transistor arrays. Figure 10 shows the functionality of the frequency and Q loops.

4. CONCLUSIONS

In this paper, we have presented an analytical technique for the solution of a class of sinusoidal oscillators. A vector lock loop, based on this class, has been proposed. The individual loops of this VLL are uncoupled. This scheme combines the best of both the VCF and VCO schemes.

5. ACKNOWLEDGEMENT

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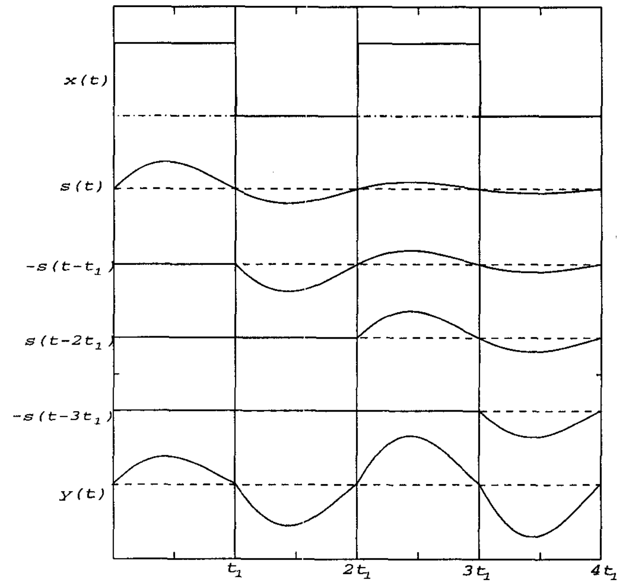


Figure 2. Oscillation buildup mechanism

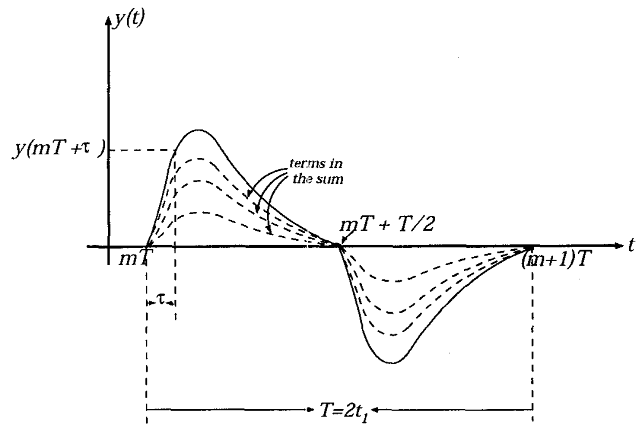
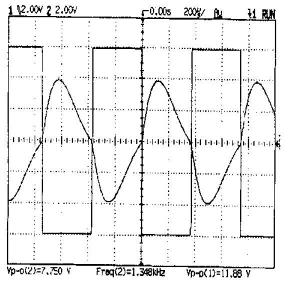
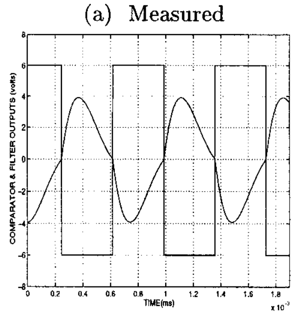


Figure 3. Timing Detail



(a) Measured



(b) Computed

Figure 4. Filter and comparator outputs

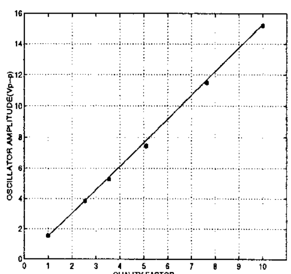


Figure 5. Measured(⊕) and predicted(—) amplitude of oscillation vs. Q

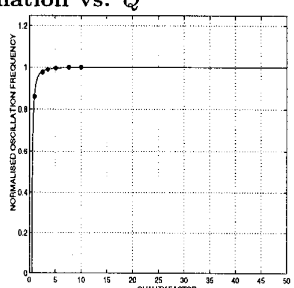


Figure 6. Measured(⊕) and predicted(—) frequency of oscillation vs. Q

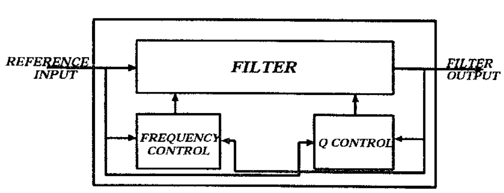
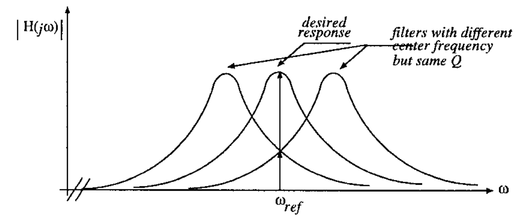
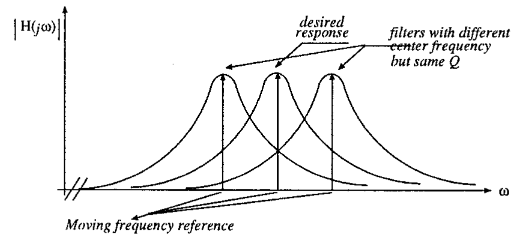


Figure 7. A conventional Vector Lock Loop(VLL)



(a) Traditional Magnitude Locked Loop



(b) Proposed Technique

Figure 8. Comparison of conventional and proposed technique

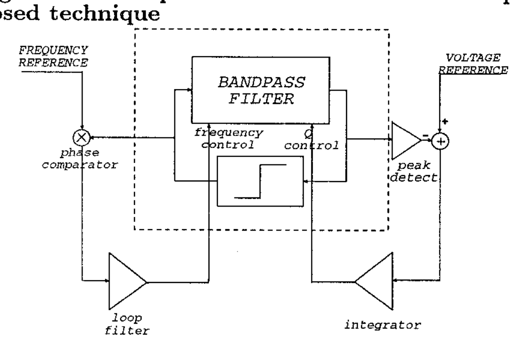
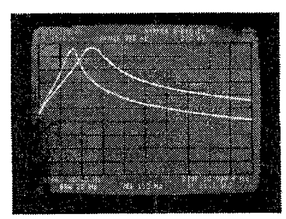
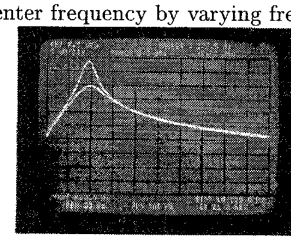


Figure 9. Proposed VLL



(a) Varying center frequency by varying frequency reference



(b) Varying quality factor by varying voltage reference

Figure 10. Functionality testing of the proposed VLL