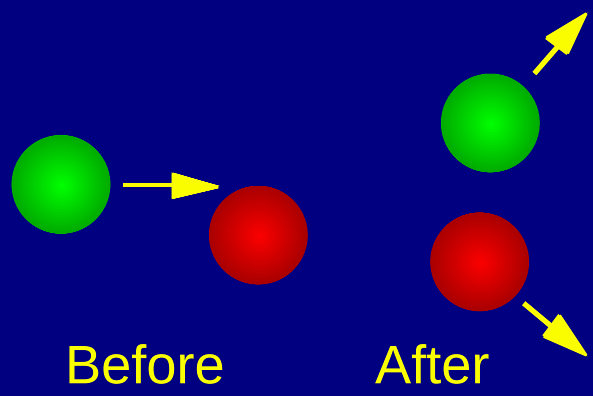
Physics

Todays Agenda

- Elastic collision in one dimension.
- Center of mass reference frame.
  - Colliding carts problem.
- Some interesting properties of elastic collisions.
  - Killer bouncing balls.

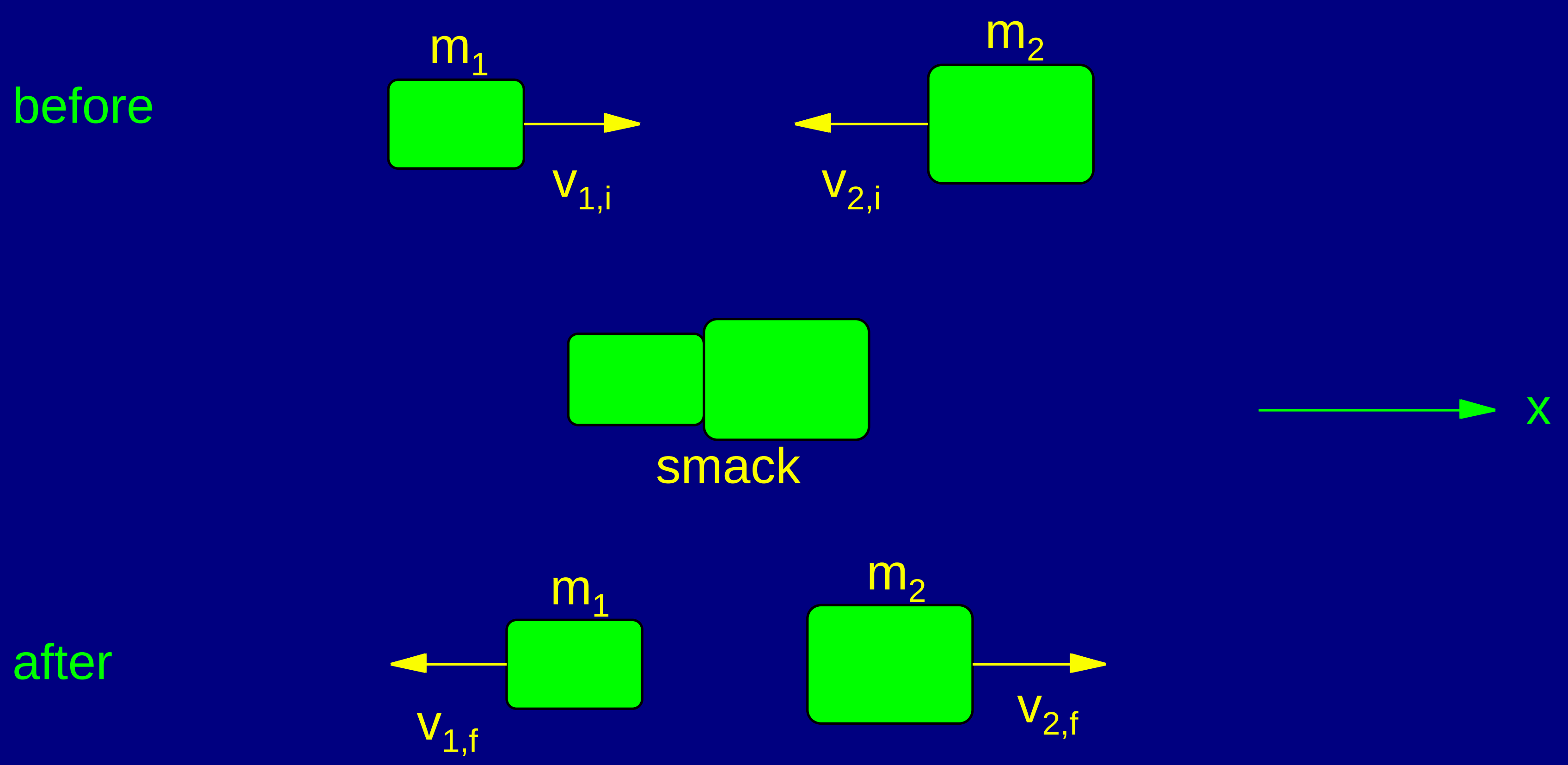
Elastic Collisions

- Elastic means energy is conserved as well as momentum.
- This gives us more constraints.
  - We can solve more complicated problems !!
  - Billiards (2-D collision).
  - The colliding objects have separate motions after the collision as well as before.

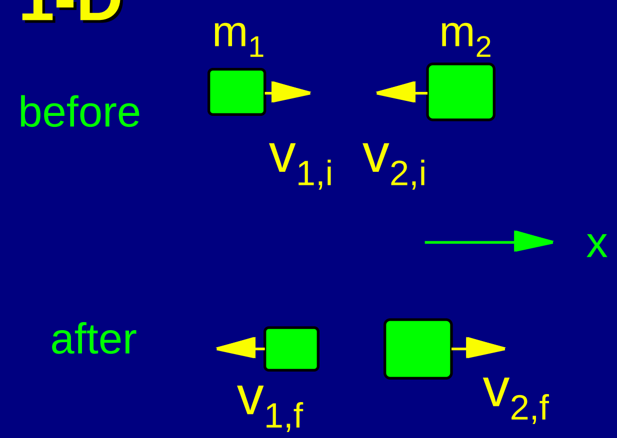


- Start with a simpler 1-D problem.

Elastic Collision in 1-D



Elastic Collision in 1-D



Conserve P_x

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

Conserve Energy

$$\tfrac{1}{2}\, m_1 v^2_{1,i} + \tfrac{1}{2}\, m_2 v^2_{2,i} = \tfrac{1}{2}\, m_1 v^2_{1,f} + \tfrac{1}{2}\, m_2 v^2_{2,f}$$

Suppose we know $v_{1,i}$ and $v_{2,i}$

We need to solve for $v_{1,f}$ and $v_{2,f}$

Should be no problem ➡ 2 equations & 2 unknowns !

Elastic Collision in 1-D

- However, solving this can sometimes get a little bit tedious since it involves a quadratic equation !!

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$\tfrac{1}{2} m_1 v^2_{1,i} + \tfrac{1}{2} m_2 v^2_{2,i} = \tfrac{1}{2} m_1 v^2_{1,f} + \tfrac{1}{2} m_2 v^2_{2,f}$$

- A simpler approach is to introduce the

Center of Mass Reference Frame:

CM Reference Frame.

- We have shown that the total momentum of a system is the velocity of the CM times the total mass:

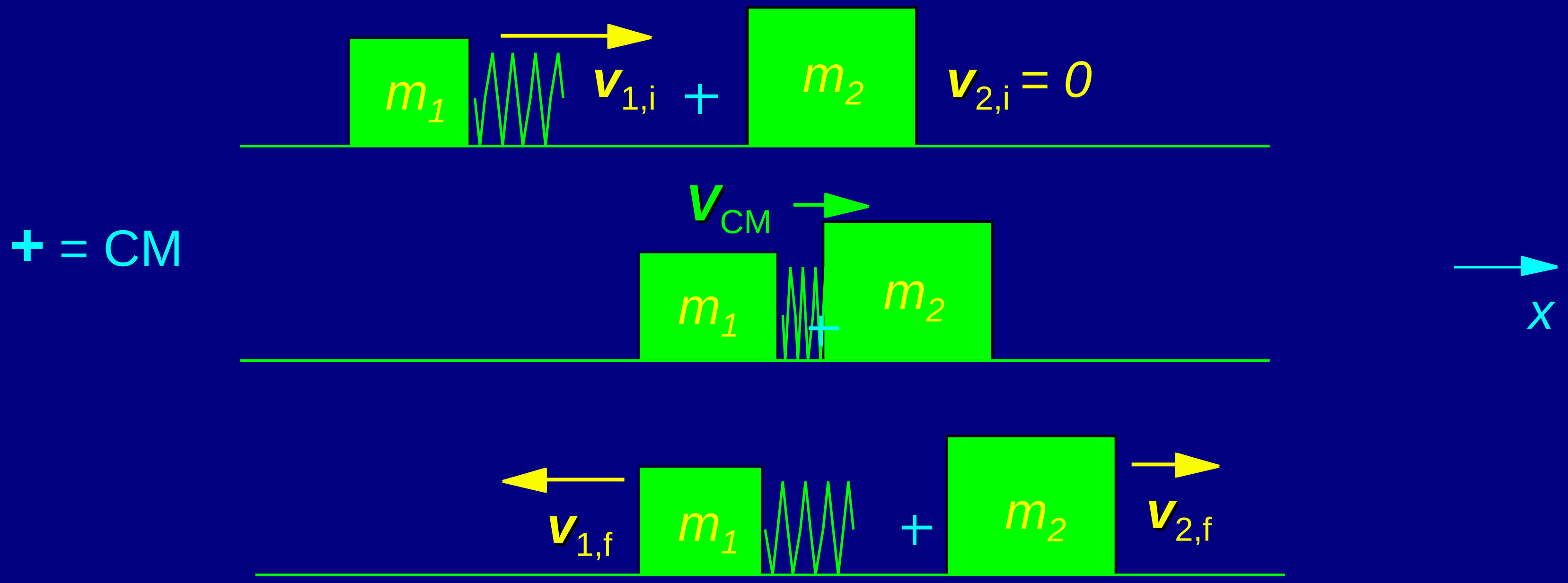
  $\boldsymbol{P}_{TOT} = M\boldsymbol{V}_{CM}$.

- We have also discussed reference frames that are related by a constant velocity vector (i.e. relative motion).

- Now consider putting yourself in a reference frame in which the CM is at rest. We call this the CM reference frame.
  - In the CM reference frame, $\boldsymbol{V}_{CM} = 0$ (by definition) and therefore $\boldsymbol{P}_{TOT} = 0$.

Example 1: Using CM Reference Frame

- A glider of mass $m_1 = .2\ kg$ slides on a frictionless track with initial velocity $v_{1,i} = 1.6\ m/s$. It hits a stationary glider of mass $m_2 = .6 kg$. A spring attached to the first glider compresses and relaxes during the collision, but there is no friction (i.e. energy is conserved). What are the final velocities ?



Example 1...

- First figure out the velocity of the CM, $\boldsymbol{V}_{\mathrm{CM}}$.

} $\boldsymbol{V}_{\mathrm{CM}} = \left(\frac{1}{m_1 + m_2}\right)(m_1 \boldsymbol{v}_{1,i} + m_2 \boldsymbol{v}_{2,i})$, but $\boldsymbol{v}_{2,i} = 0$ so

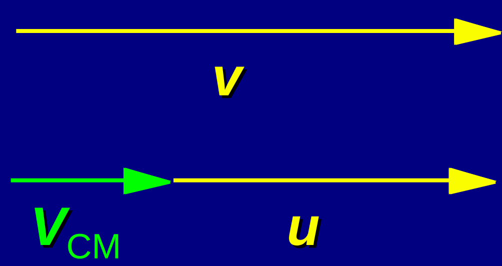
$$\boxed{\boldsymbol{V}_{\mathrm{CM}} = \left(\frac{m_1}{m_1 + m_2}\right)\boldsymbol{v}_{1,i}}$$

- So $\boldsymbol{V}_{\mathrm{CM}} = {}^{1}/_{4}\ (1.6\ m/s) = 0.4\ m/s$

Example 1...

- If the velocity of the CM in the “lab” reference frame is $\boldsymbol{V}_{CM}$, and the velocity of some particle in the “lab” reference frame is $\boldsymbol{v}$, then the velocity of the particle in the CM reference frame is $\boldsymbol{u}$:

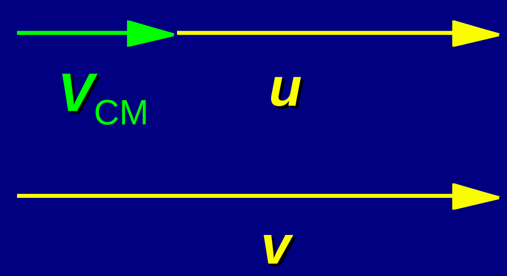
$\boldsymbol{u} = \boldsymbol{v} - \boldsymbol{V}_{CM}$ (where $\boldsymbol{u}$ $\boldsymbol{v}$ $\boldsymbol{V}_{CM}$ are vectors)



Example 1...

- Similarly, if you know ***u***, you can find ***v*** by relating the CM reference frame back to the lab reference frame:

  - since $\boldsymbol{u} = \boldsymbol{v} - \boldsymbol{V}_{CM}$ then $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{V}_{CM}$



Example 1 (aside on energy)

- Consider the total energy of the system in the LAB reference frame:

$$E_{LAB} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

but

$$\mathbf{v}_1 = \mathbf{V}_{CM} + \mathbf{u}_1$$
$$\mathbf{v}_2 = \mathbf{V}_{CM} + \mathbf{u}_2$$

so

$$v_1^2 = \mathbf{v}_1 \cdot \mathbf{v}_1 = V_{CM}^2 + u_1^2 + 2\mathbf{V}_{CM} \cdot \mathbf{u}_1$$

(same for v_2)

$$E_{LAB} = \underbrace{\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2}_{= K_{REL}} + \underbrace{\frac{1}{2}(m_1 + m_2)V_{CM}^2}_{= K_{CM}} + \mathbf{V}_{CM} \cdot \underbrace{(m_1\mathbf{u}_1 + m_2\mathbf{u}_2)}_{= P_{TOT,CM} = 0}$$

Example 1 (aside on energy)...

- Consider the total energy of the system in the LAB reference frame:

$$E_{LAB} = \underbrace{\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2}_{= K_{REL}} + \underbrace{\frac{1}{2}(m_1 + m_2)V_{CM}^2}_{= K_{CM}}$$

So $E_{LAB} = K_{REL} + K_{CM}$

K_{CM} is the kinetic energy of the center of mass.

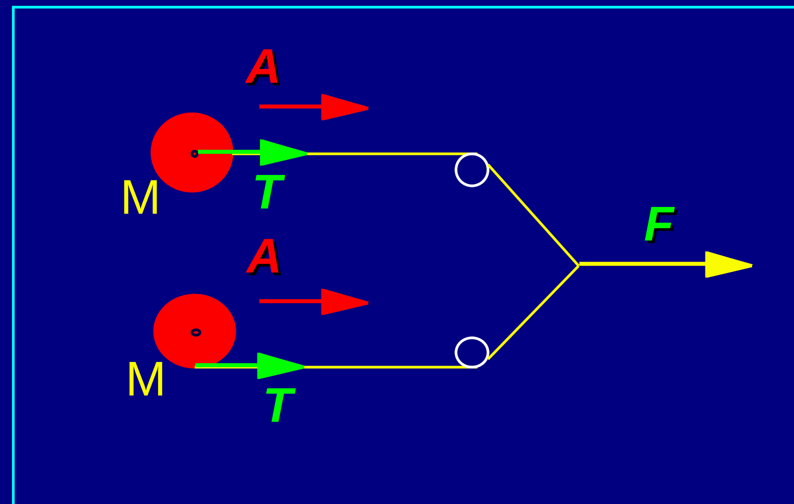
K_{REL} is the kinetic energy due to “relative” motion in the CM frame.

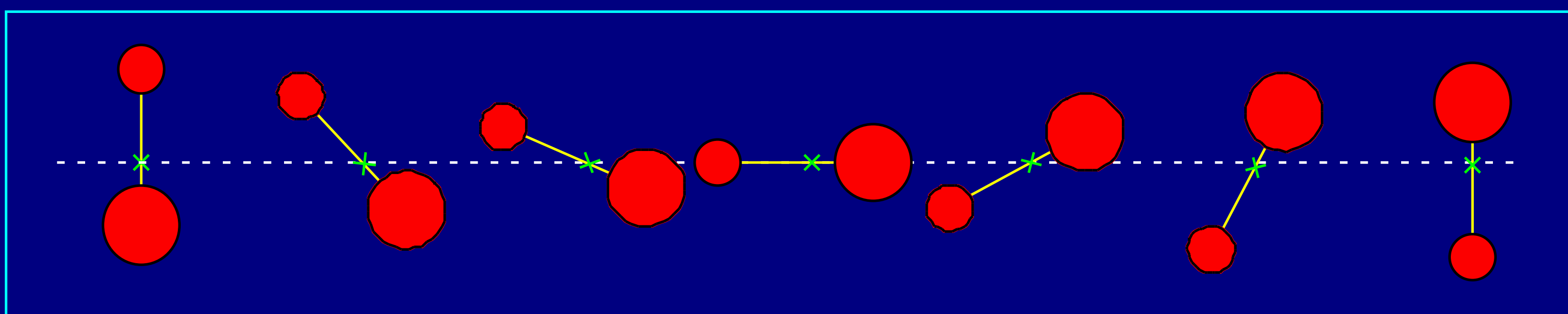
This is true in general, not just 1-D

Example 1 (aside on energy)...

$E_{LAB} = K_{REL} + K_{CM}$

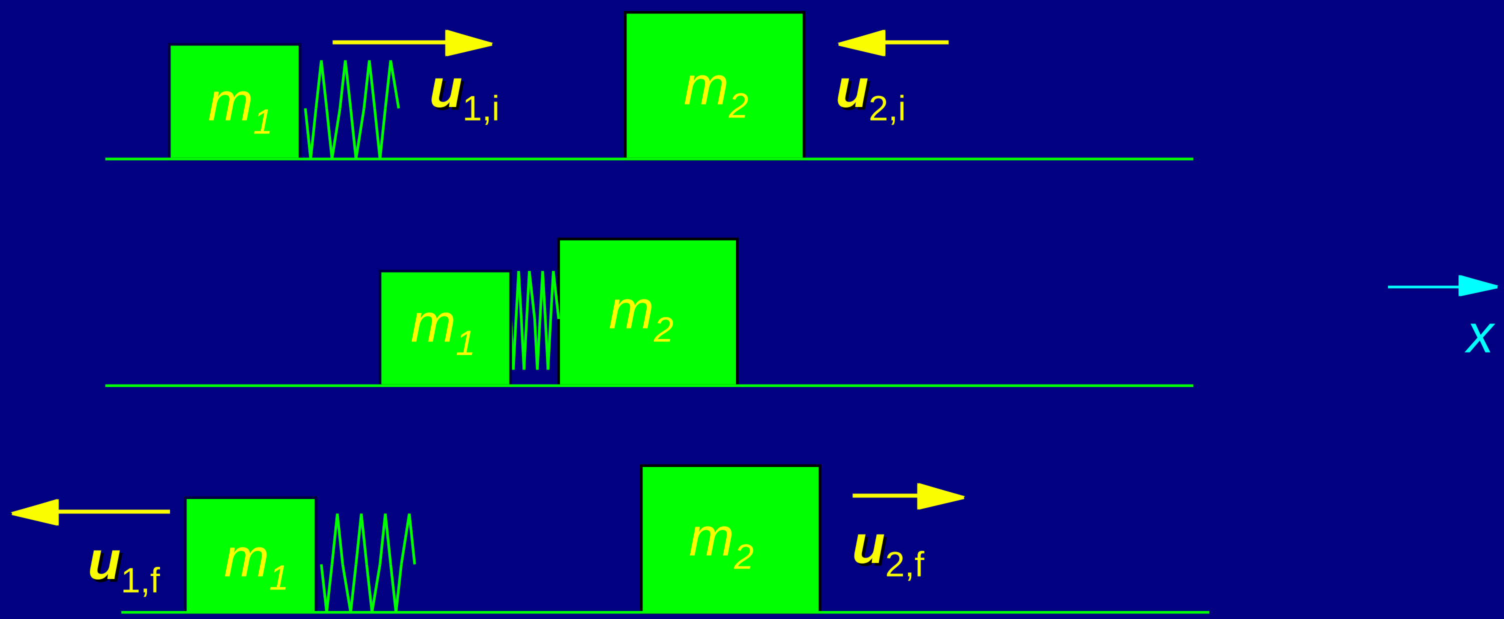
- Does total energy depend on the reference frame ??
- YOU BET !





Example 1 continued...

- Now consider the collision viewed from a frame moving with the CM velocity V_{CM}. (jargon: "in the CM frame")



Example 1...

- Calculate the initial velocities in the CM reference frame (all velocities in *x* direction):

$$\boldsymbol{u}_{1,i} = \boldsymbol{v}_{1,i} - \boldsymbol{V}_{CM} = 1.6 \text{ m/s} - 0.4 \text{ m/s} = 1.2 \text{ m/s}$$

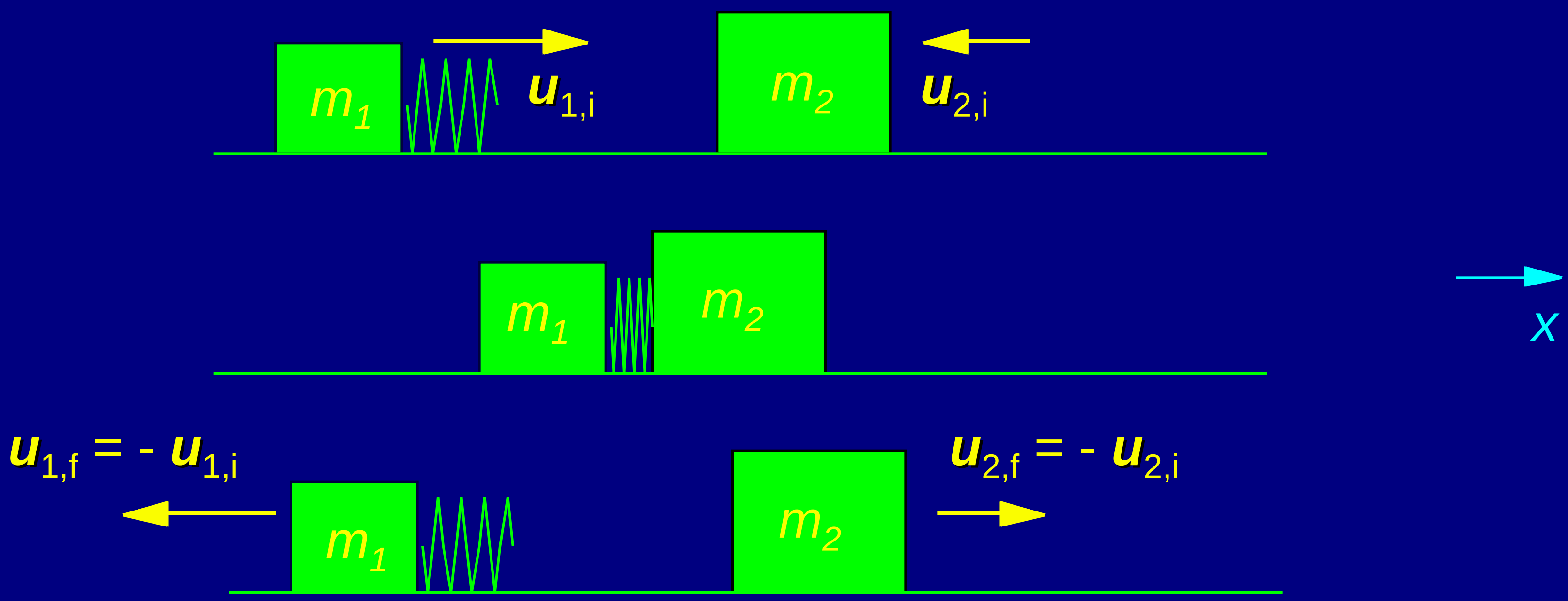
$$\boldsymbol{u}_{2,i} = \boldsymbol{v}_{2,i} - \boldsymbol{V}_{CM} = 0 \text{ m/s} - 0.4 \text{ m/s} = -0.4 \text{ m/s}$$

$$\boldsymbol{u}_{1,i} = 1.2 \text{ m/s}$$

$$\boldsymbol{u}_{2,i} = -0.4 \text{ m/s}$$

Example 1...

- Now that we know $\boldsymbol{u}_{1,i}$ and $\boldsymbol{u}_{2,i}$, lets figure out the final velocities in the CM reference frame, $\boldsymbol{u}_{1,f}$ and $\boldsymbol{u}_{2,f}$.



Example 1...

- Use energy conservation to relate initial and final velocities.
- The total energy in the CM frame before and after the collision is the same:

$$\frac{1}{2}m_1u_{1,i}^2 + \frac{1}{2}m_2u_{2,i}^2 = \frac{1}{2}m_1u_{1,f}^2 + \frac{1}{2}m_2u_{2,f}^2$$

- Using the fact that the total momentum in the CM frame is zero, we use the above find that the speed of a particle in the CM frame is the same before and after a collision.

$$\boxed{u_{1,i}^2 = u_{1,f}^2} \quad \text{and} \quad \boxed{u_{2,i}^2 = u_{2,f}^2}$$

- This is true in general. In 1-D this gives the simple result:

$$u_{1,f} = -u_{1,i} \qquad \text{and} \qquad u_{2,f} = -u_{2,i}$$

Example 1...

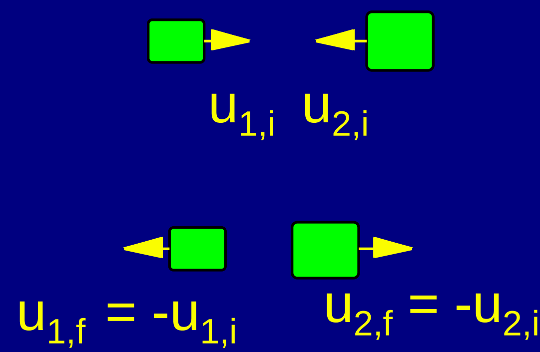
- So now we can calculate the initial velocities in the lab reference frame:

$$\boldsymbol{v}_{1,f} = \boldsymbol{u}_{1,f} + \boldsymbol{V}_{CM} = -1.2 \text{ m/s} + 0.4 \text{ m/s} = -0.8 \text{ m/s}$$

$$\boldsymbol{v}_{2,f} = \boldsymbol{u}_{2,f} + \boldsymbol{V}_{CM} = 0.4 \text{ m/s} + 0.4 \text{ m/s} = 0.8 \text{ m/s}$$

$$\boxed{\begin{array}{l} \boldsymbol{v}_{1,f} = -0.8 \text{ m/s} \\ \boldsymbol{v}_{2,f} = 0.8 \text{ m/s} \end{array}}$$

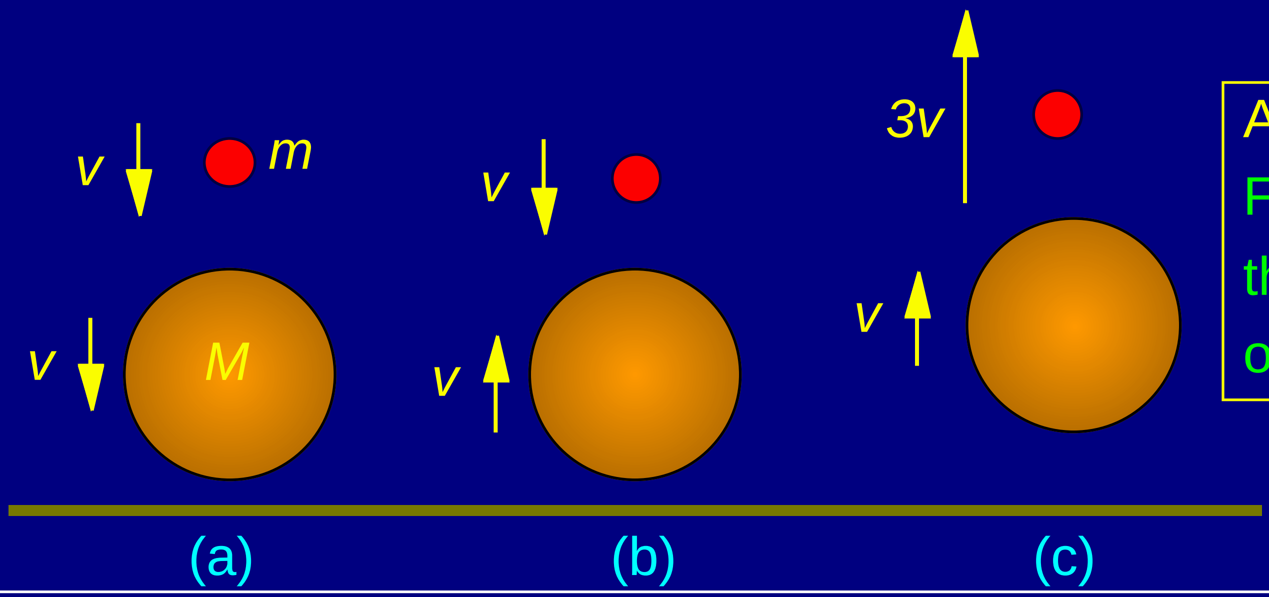
Interesting Fact



- We just showed that in the CM reference frame the speed of an object is the same before and after the collision, although the direction changes.
- The relative speed if the blocks is therefore equal and opposite before and after the collision.
  $(u_{1,i} - u_{2,i}) = -(u_{1,f} - u_{2,f})$
- But since the measurement of a difference of speeds does not depend on reference frame, we can say that
  The relative speed if the blocks is therefore equal and opposite before and after the collision, **in any reference frame.**
  - Rate of approach = rate of recession.

Basketball Demo.

- Carefully place a small rubber ball (mass *m*) on top of a much bigger basketball (mass *M*). Drop these from some height. The height reached by the small ball of the after they "bounce" is ~ 9 times the original height !! (Assumes $M >> m$ and all bounces are elastic).
  - Understand this using the "speed of approach = speed of recession" property we just proved.



Assignment: Figure out the factor of *9*.

VHS-Physics-SDW

Recap

- Elastic collision in one dimension.
- Center of mass reference frame.
  - Colliding carts problem.
- Some interesting properties of elastic collisions.
  - Killer bouncing balls
- Work the problems