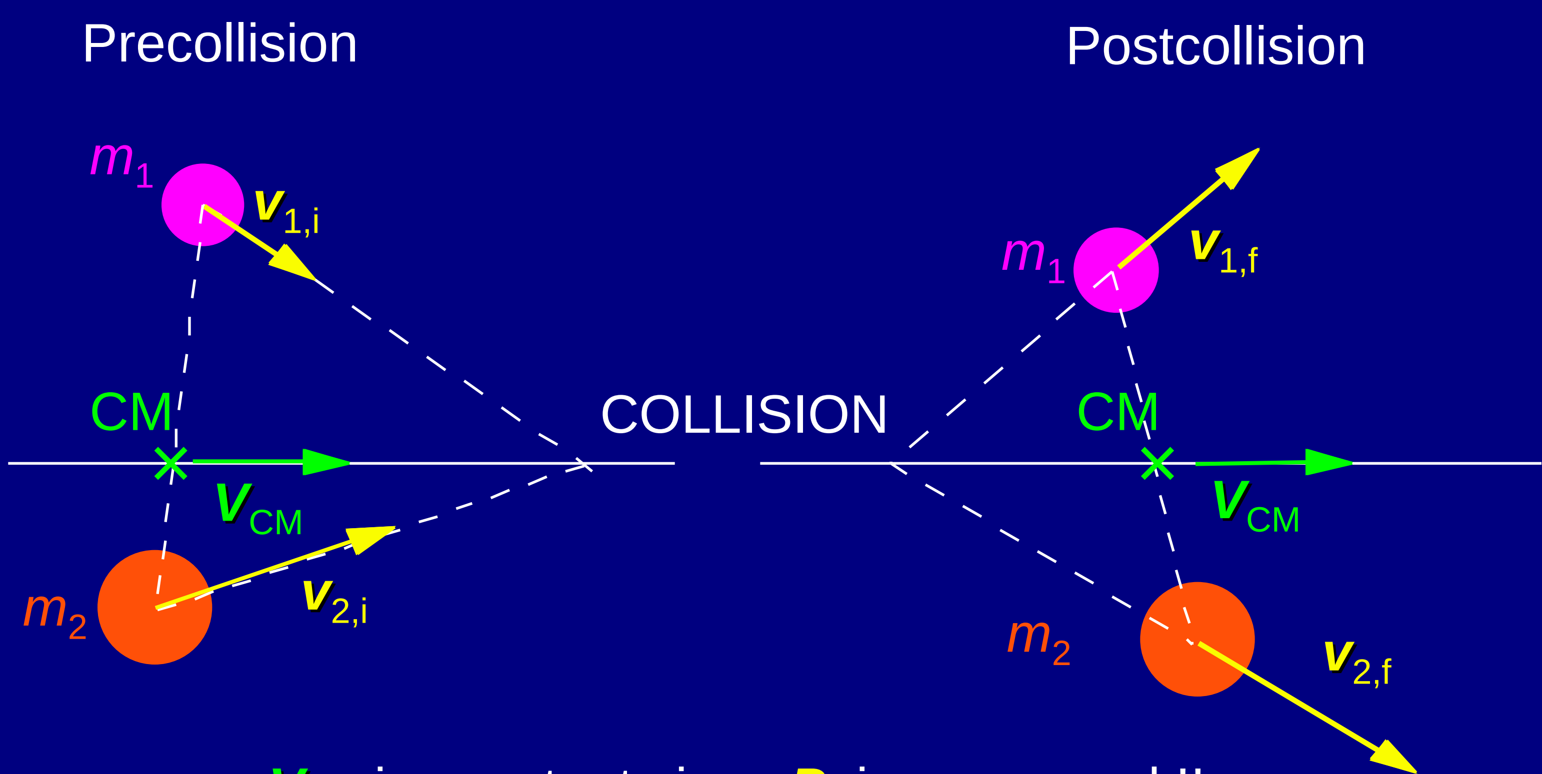
Physics

Todays Agenda

- Two-dimensional elastic collisions.
- Examples (nuclear scattering, billiards).
- Impulse and average force.

2-D Elastic Collision of 2 objects



$\boldsymbol{V}_{CM}$ is constant since $\boldsymbol{P}$ is conserved !!

Energy in Elastic Collisions:

- Use energy conservation to relate initial and final velocities.
- The total energy in the CM frame before and after the collision is the same:

$$\frac{1}{2m_1}m_1^2u_{1,i}^2 + \frac{1}{2m_2}m_2^2u_{2,i}^2 = \frac{1}{2m_1}m_1^2u_{1,f}^2 + \frac{1}{2m_2}m_2^2u_{2,f}^2$$

- But the total momentum is zero: $\Rightarrow$ $(m_1u_{1,i})^2 = (m_2u_{2,i})^2$
- So: $\left(\frac{1}{2m_1} + \frac{1}{2m_2}\right)m_1^2u_{1,i}^2 = \left(\frac{1}{2m_1} + \frac{1}{2m_2}\right)m_1^2u_{1,f}^2$

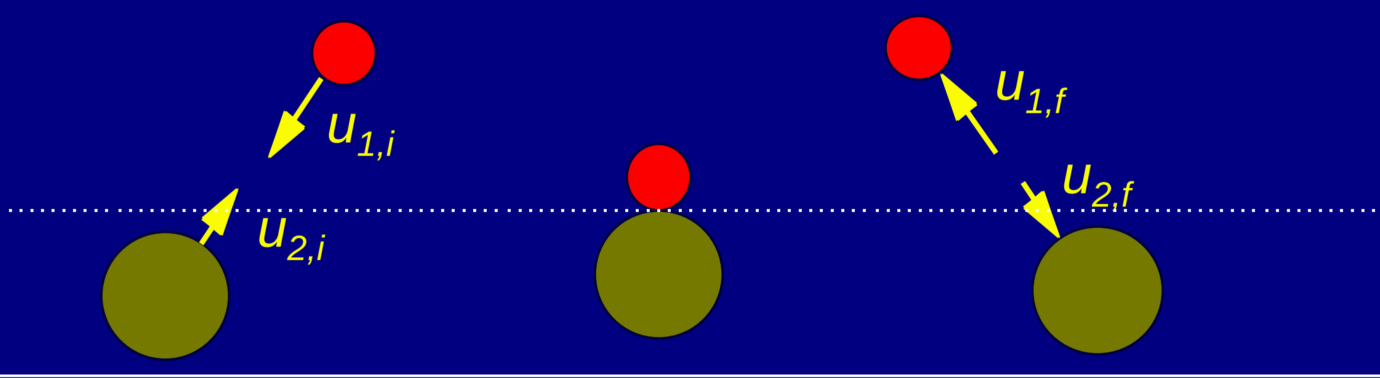
$$\Rightarrow \boxed{u_{1,i}^2 = u_{1,f}^2}$$

(and the same for particle 2)

Elastic Collisions:

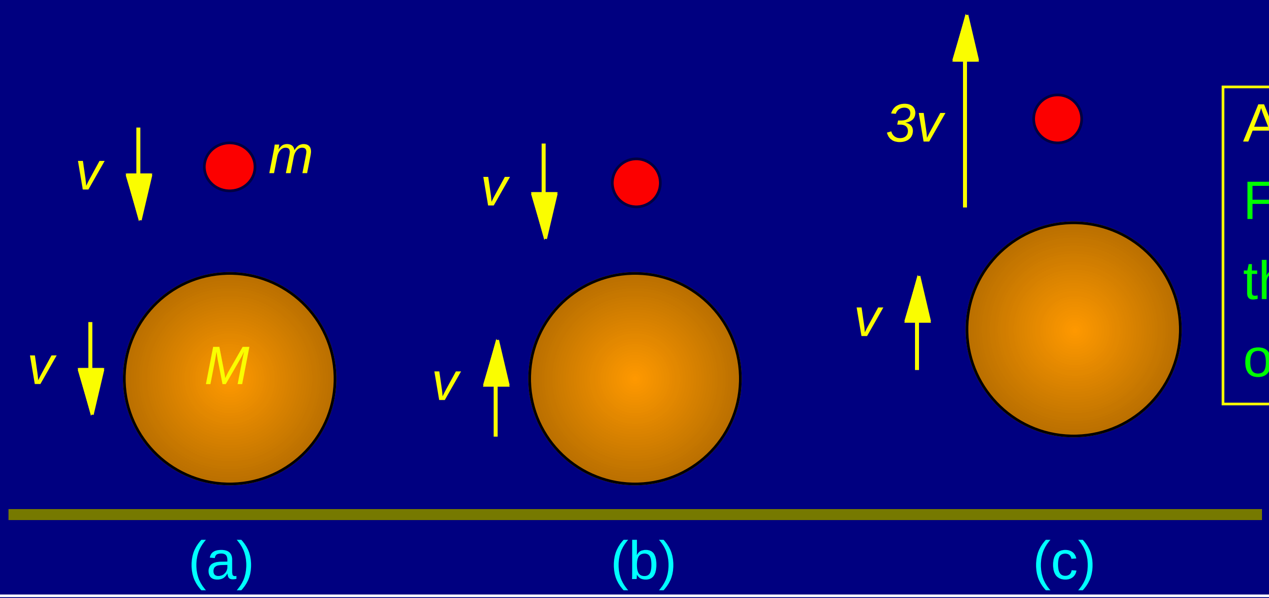
- So we see that: $u_{1,i}^2 = u_{1,f}^2 \quad u_{2,i}^2 = u_{2,f}^2$
  - The speed of a particle as measured in the CM reference frame is not changed by an elastic collision.
  - The velocity is changed in general, since directions change.

- Since the speed of each particle is unchanged, the sum of these speeds is also unchanged.
  - speed of approach = speed of recession



Basketball Demo.

- Carefully place a small rubber ball (mass *m*) on top of a much bigger basketball (mass *M*). Drop these from some height. The height reached by the small ball of the after they "bounce" is ~ 9 times the original height !! (Assumes $M >> m$ and all bounces are elastic).
  - Understand this using the "speed of approach = speed of recession" property we just proved.



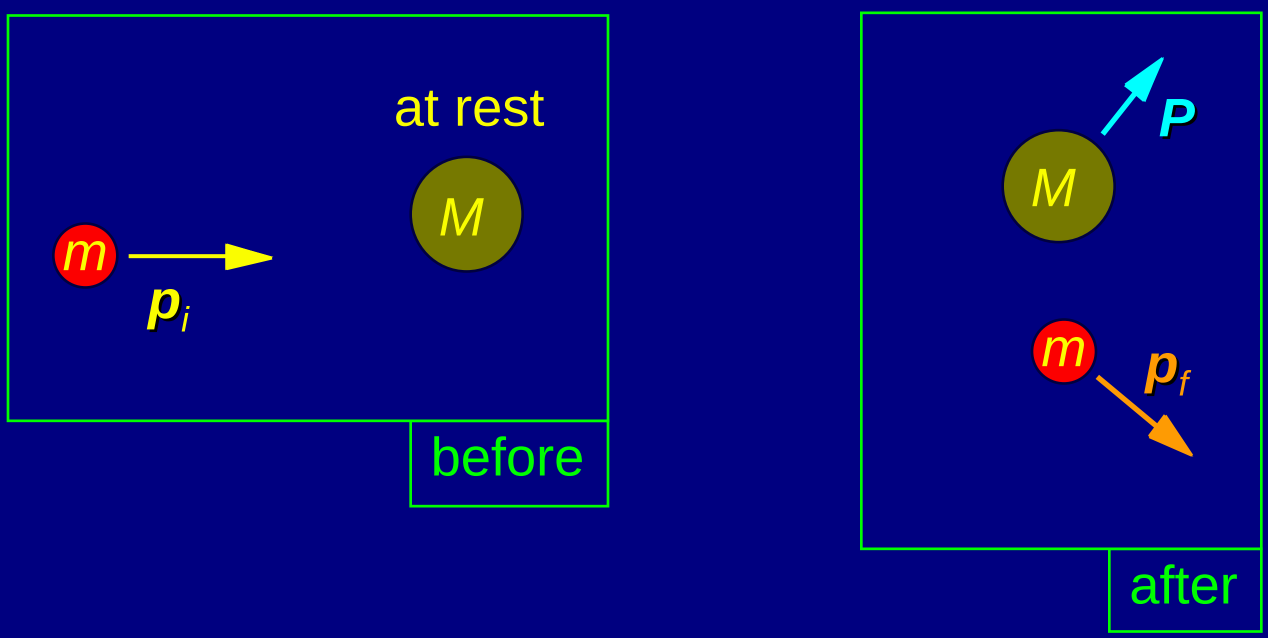
Assignment:
Figure out the factor of *9*.

2-D Elastic Collision of 2 objects

- Suppose we know what the "precollision" velocities are.
- We want to find out about the motion of both objects after the collision.
  - We want $v_{1x,\mathrm{f}}$, $v_{1y,\mathrm{f}}$, $v_{2x,\mathrm{f}}$, $v_{2y,\mathrm{f}}$
- What else do we know :
  - In an elastic collision, energy is conserved as well as momentum. This leads to 3 equations:
  - $E_\mathrm{f} = E_\mathrm{i}$
  - $P_{\mathrm{x,f}} = P_{\mathrm{x,i}}$ (where $P_\mathrm{x} = p_{1x} + p_{2x} = m_1 v_{1x} + m_2 v_{2x}$ etc)
  - $P_{\mathrm{y,f}} = P_{\mathrm{y,i}}$
- We have 3 equations and 4 unknowns:
  - In general we need more information !!
  - for example...

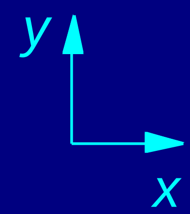
2-D Elastic Collision: Nuclear Scattering

- A particle of unknown mass M is initially at rest. A particle of known mass m is "shot" at it with initial momentum $\boldsymbol{p}_i$. After the particles collide, the new momentum of the shot particle $\boldsymbol{p}_f$ is measured.
  - Figure out what M is in terms of $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$ and m.

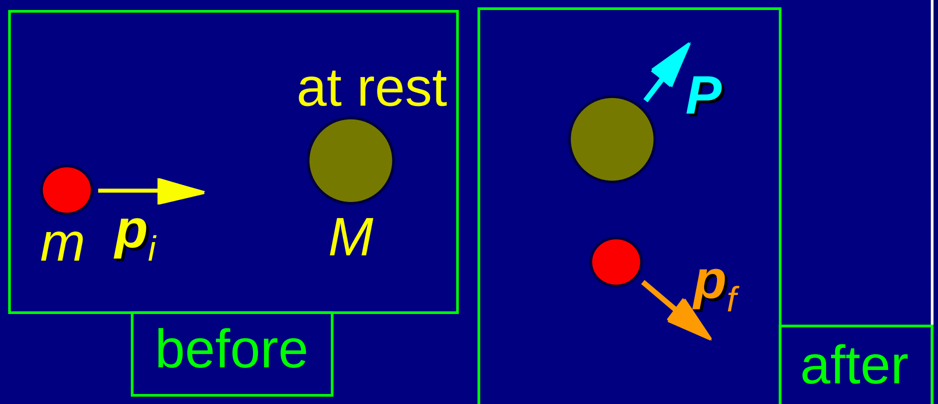


2-D Elastic Collision: Nuclear Scattering

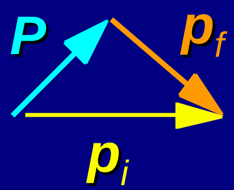
- Conserve momentum:
  - $p_{x,i} = P_x + p_{x,f}$ (1)
  - $p_{y,i} = 0 = P_y + p_{y,f}$ (2)



- Conserve energy: (recall $K = {}^1/_2 mv^2 = p^2/2m$)
  - $p_i^2 / 2m = p_f^2 / 2m + P^2 / 2M$ (3)
- We have 3 equations
  - (1) (2) (3)
- We have 3 unknowns
  - P_x, P_y, M
- We should be all set !



2-D Elastic Collision: Nuclear Scattering



- Using momentum conservation: $\boldsymbol{p}_i = \boldsymbol{p}_f + \boldsymbol{P}$
  - So $P^2 = (\boldsymbol{p}_i - \boldsymbol{p}_f)^2$
- Using energy conservation:

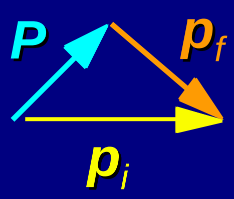
$$\frac{p_i^2}{2m} = \frac{p_f^2}{2m} + \frac{P^2}{2M} \quad \Rightarrow \quad P^2 = 2M\left(\frac{p_i^2}{2m} - \frac{p_f^2}{2m}\right)$$

and using $P^2 = (\boldsymbol{p}_i - \boldsymbol{p}_f)^2$

$$\Rightarrow \quad \boxed{M = m\left[\frac{(\boldsymbol{p}_i - \boldsymbol{p}_f)^2}{p_i^2 - p_f^2}\right]}$$

2-D Elastic Collision: Nuclear Scattering

- So we find that $M = m\left[\frac{(\boldsymbol{p}_i - \boldsymbol{p}_f)^2}{p_i^2 - p_f^2}\right]$

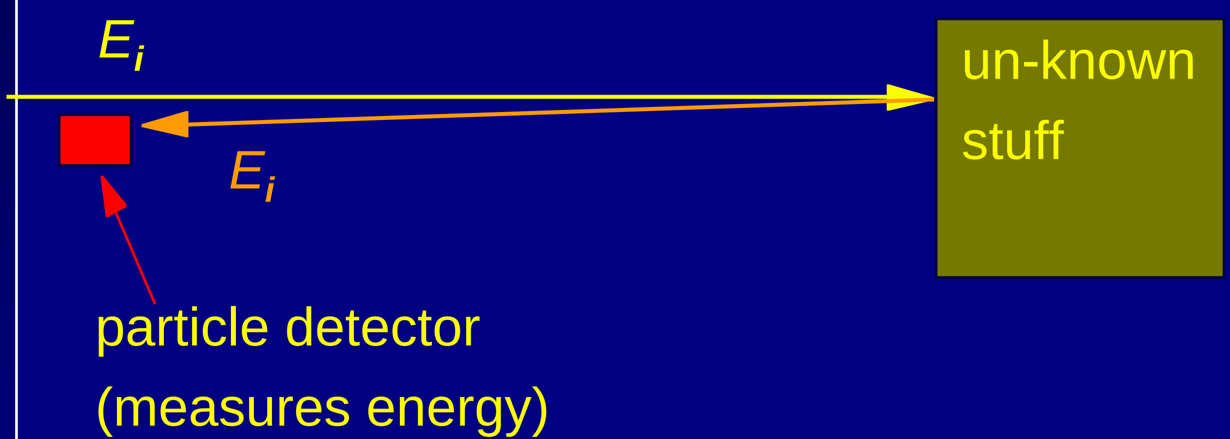


- If we measure $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$ and we know m we can measure M.
  - We can learn about something we cant see !

- This is the basic idea behind a large body of work done in atomic, nuclear and particle physics.

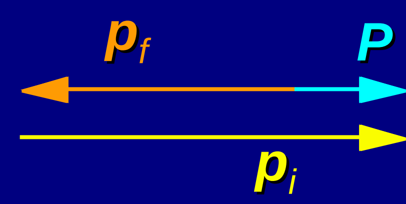
Rutherford Backscattering

- Shoot a beam of α particles (helium nuclei) having known energy E_i into a sample of unknown composition. Measure the energy E_f of the α particles that bounce back out at ~180^o with respect to the incoming beam.



Rutherford Backscattering

- In the 180^{o} case $M = m\left[\frac{(\boldsymbol{p}_i - \boldsymbol{p}_f)^2}{p_i^2 - p_f^2}\right]$



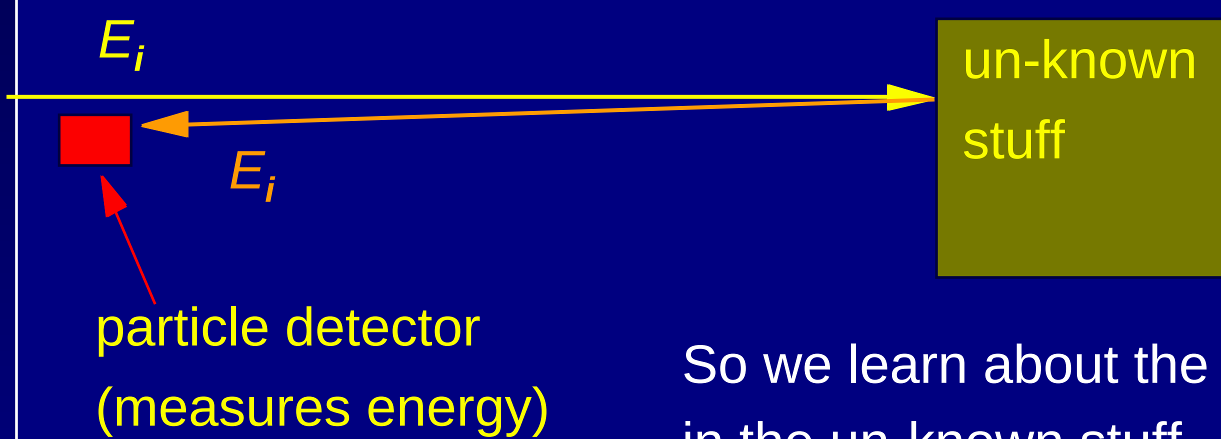
simplifies signifiantly: $M = m\left[\frac{(p_i + p_f)^2}{p_i^2 - p_f^2}\right] = \frac{(v_i + v_f)(v_i + v_f)}{(v_i + v_f)(v_i - v_f)}$

$\Rightarrow$ $M = m\frac{(v_i + v_f)}{(v_i - v_f)}$ $\Rightarrow$ $v_f = v_i\frac{(M - m)}{(M + m)}$

$\Rightarrow$ $$\boxed{\frac{E_f}{E_i} = \left(\frac{M - m}{M + m}\right)^2}$$

Rutherford Backscattering

- Shoot a beam of α particles (helium nuclei) having known energy E_i into a sample of unknown composition. Measure the energy E_f of the α particles that bounce back out at ~180^o with respect to the incoming beam.



$$\frac{E_f}{E_i} = \left(\frac{M - m}{M + m}\right)^2$$

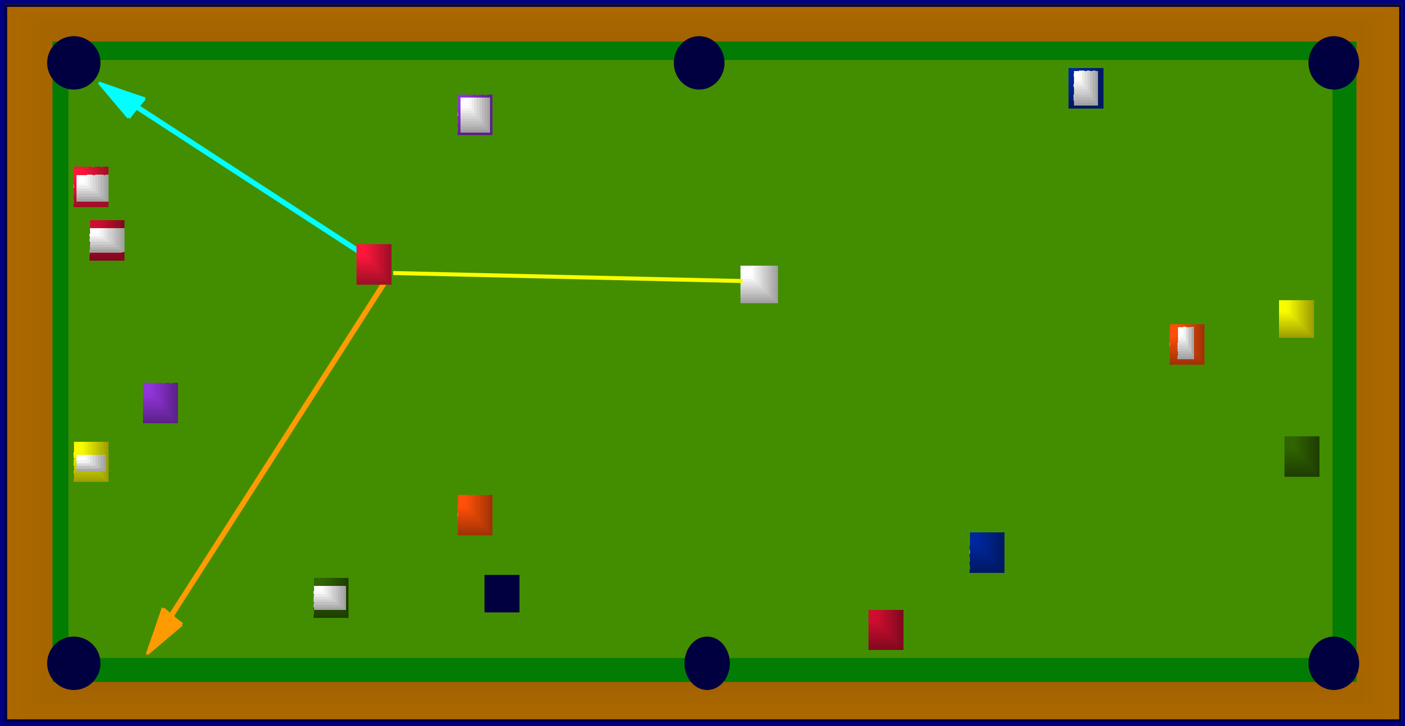
So we learn about the mass of the nuclei in the un-known stuff. (We learn what the stuff is).

Rutherford Backscattering

- For example: Suppose we are shooting α particles that have an initial energy of $E_i = 2\ MeV$ at a target made of Aluminum.
  - $m(\alpha) = 4$
  - $M(Al) = 27$
- So $E_f = E_i \left(\dfrac{M - m}{M + m} \right)^2 = (2MeV)\left(\dfrac{23}{31} \right)^2 = 1.1 MeV$
-

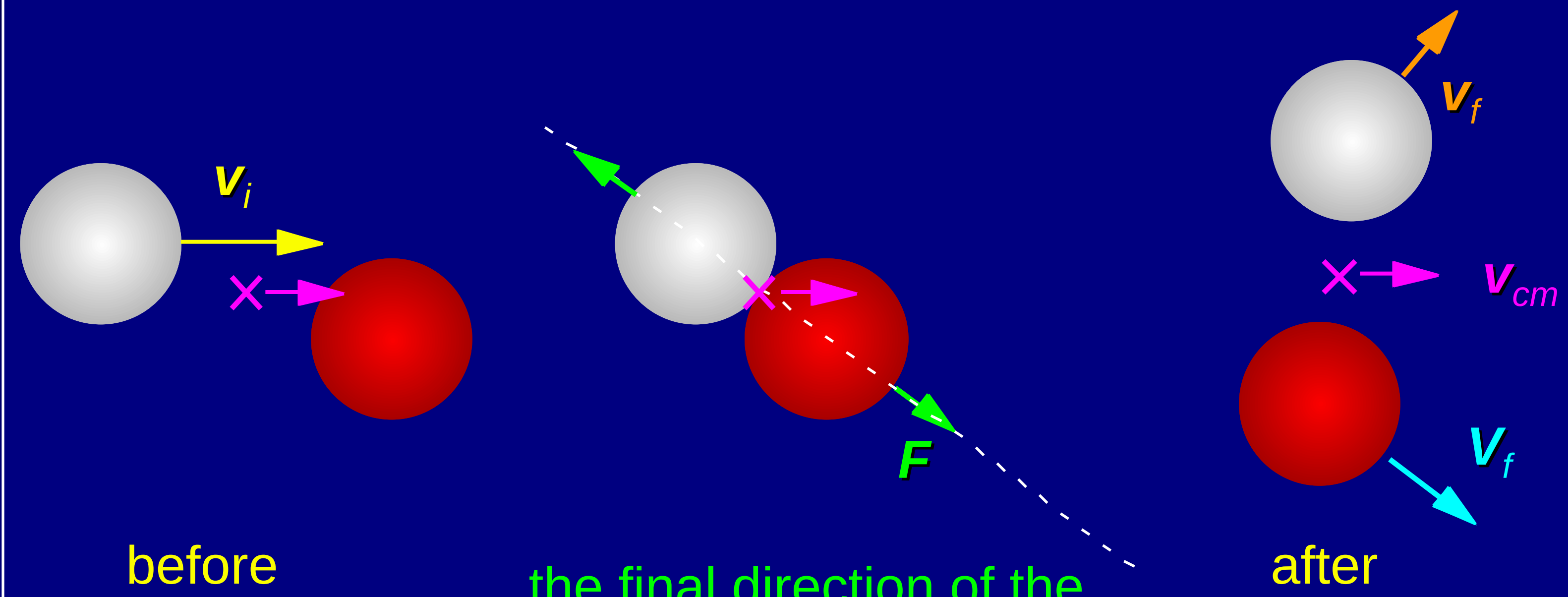
Another example of 2-D elastic collisions: Billiards.

- If all we know is the initial velocity of the cue ball, we dont have enough information to solve for the exact paths after the collision. But we can learn some useful things...



Billiards.

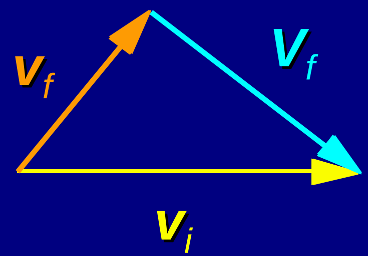
- Consider the case where one ball is initially at rest.



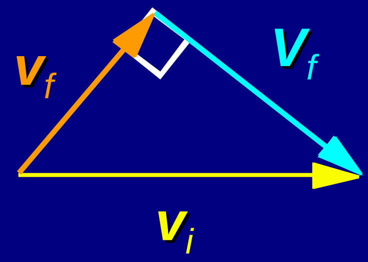
the final direction of the red ball will depend on where the balls hit.

Billiards.

- We know momentum is conserved: $m\mathbf{v}_i = m\mathbf{v}_f + m\mathbf{V}_f$
- Since the masses of the balls are all the same:
  - $\mathbf{v}_i = \mathbf{v}_f + \mathbf{V}_f$

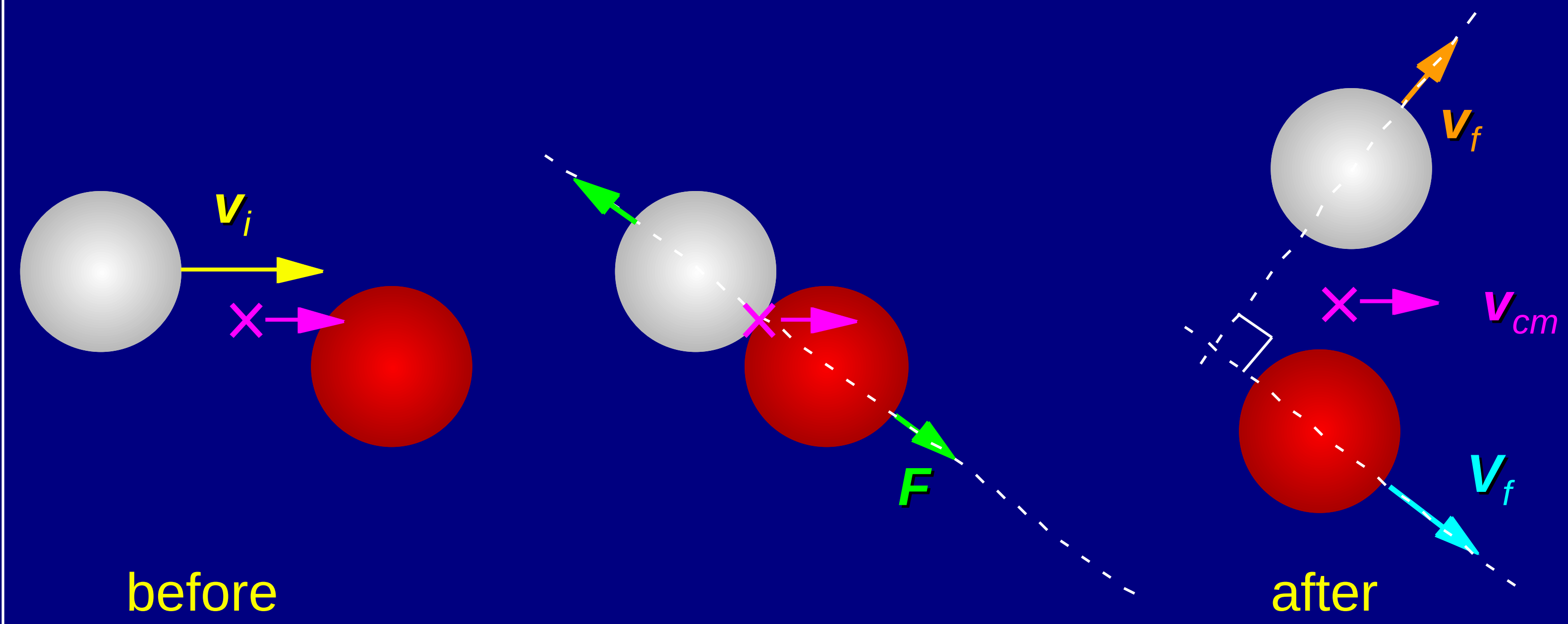


- We also know energy is conserved: $\frac{1}{2}m\mathbf{v}_i^2 = \frac{1}{2}m\mathbf{v}_f^2 + \frac{1}{2}m\mathbf{V}_f^2$
  - $\mathbf{v}_i^2 = \mathbf{v}_f^2 + \mathbf{V}_f^2$
  - Pythagoras equation !
  - $\mathbf{v}_f^2 + \mathbf{V}_f^2$ must be orthogonal !



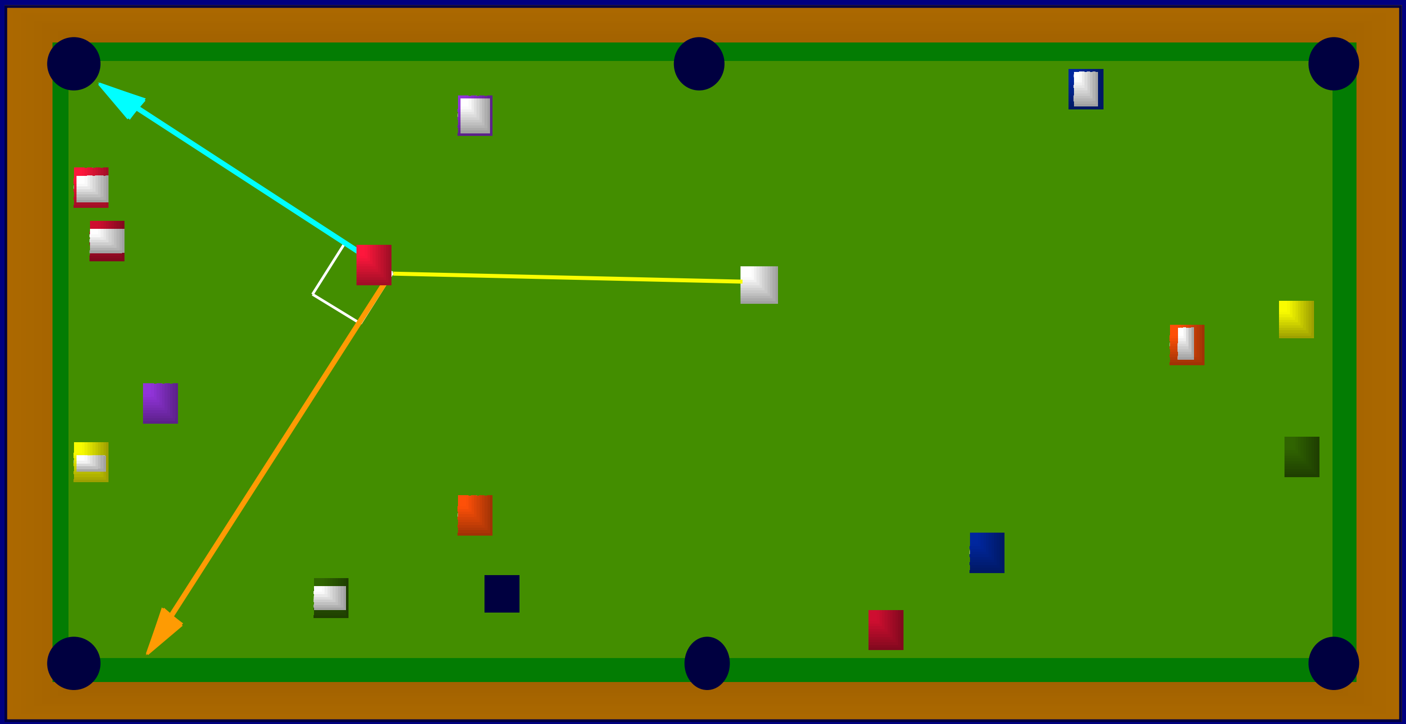
Billiards.

- The final directions are separated by 90º.



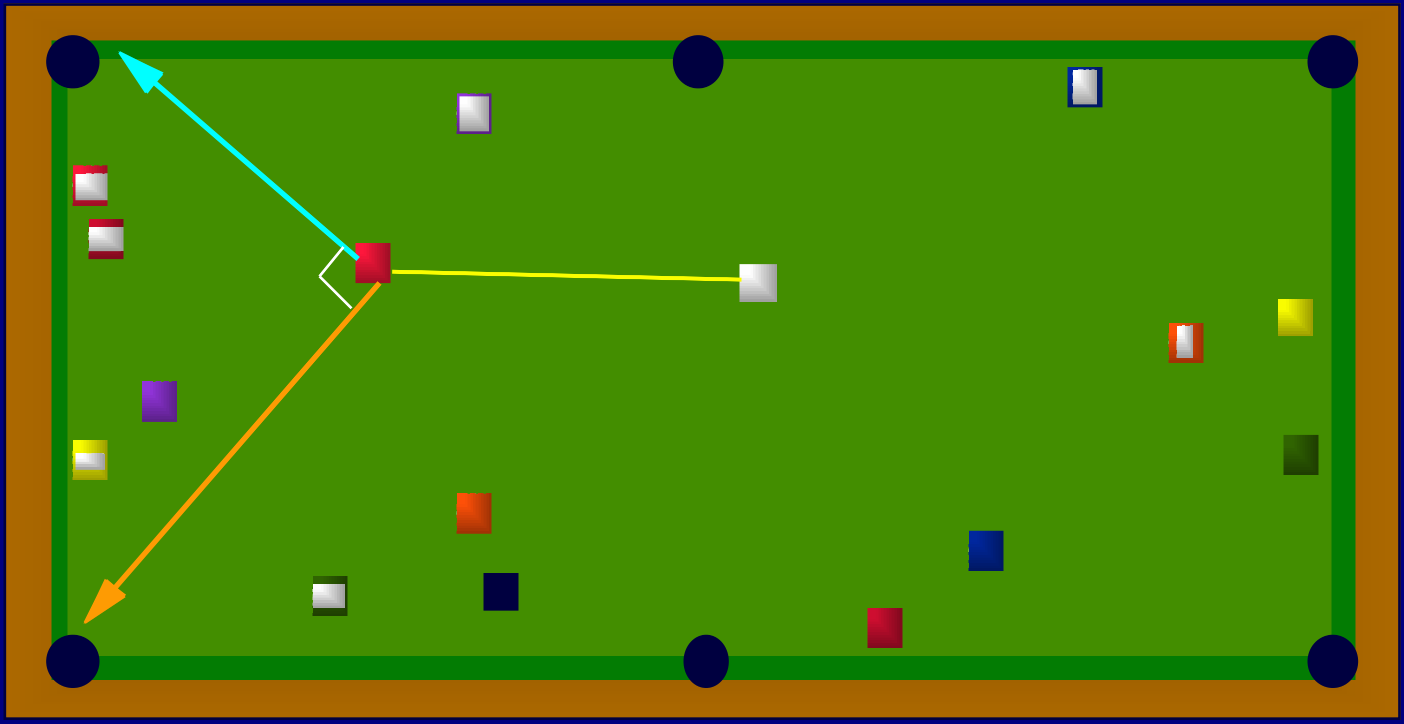
Billiards.

- So, we can sink the red ball without sinking the white ball.



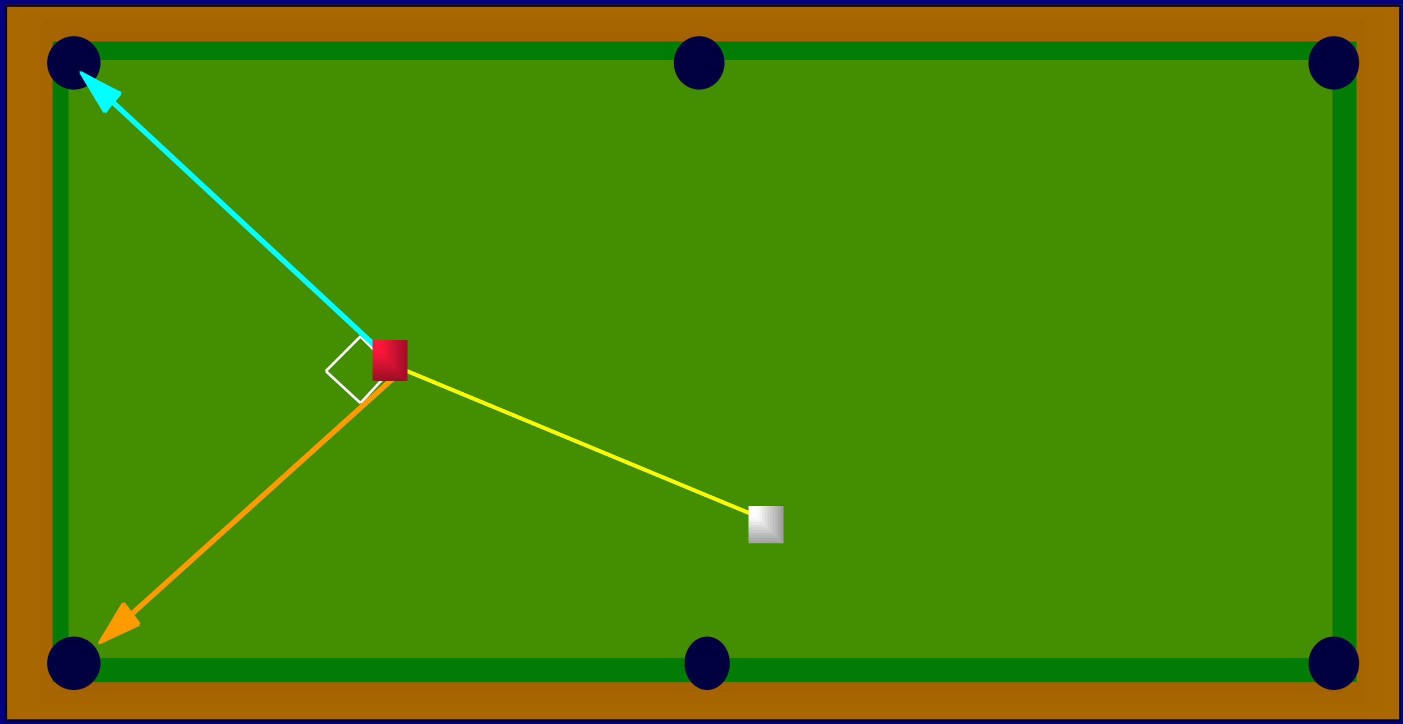
Billiards.

- So, we can sink the red ball without sinking the white ball.
- However, we can also miss. All we know is that the angle between the balls is 90º.



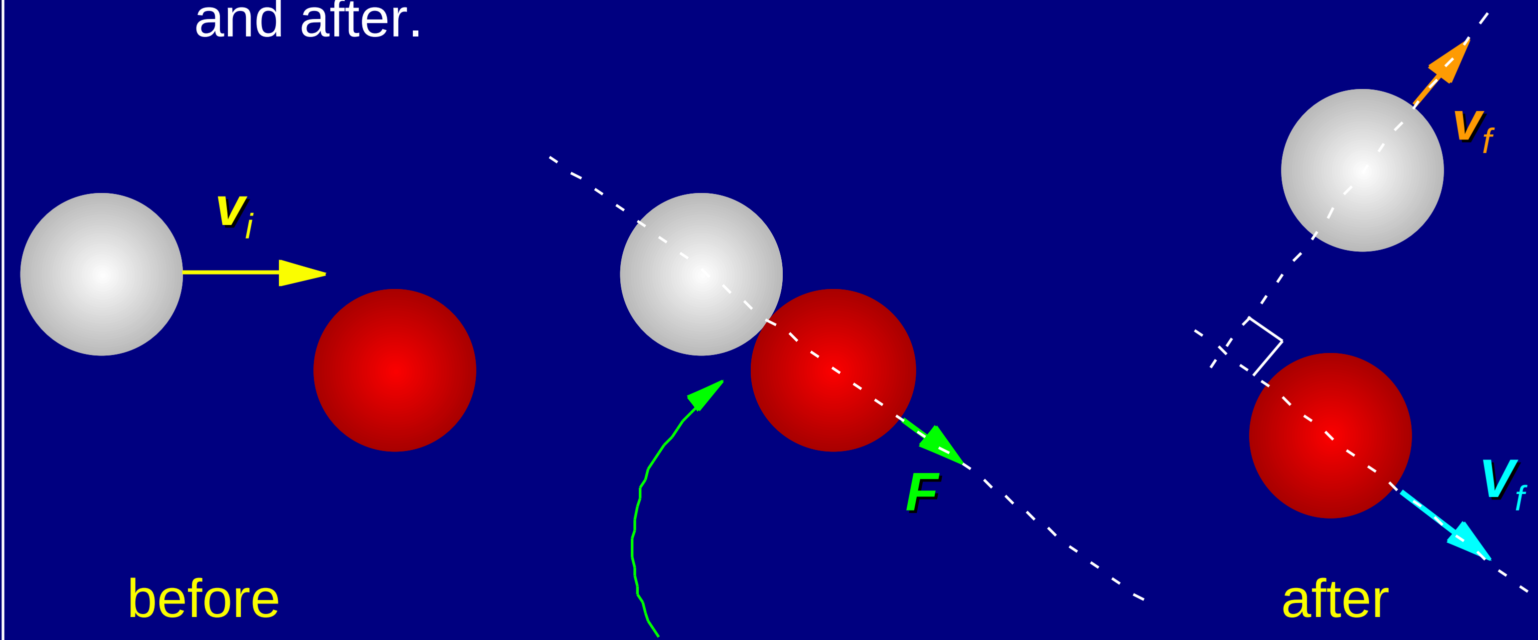
Billiards.

- **Tip:** If you shoot a ball spotted on the "dot", the inclination will be to sink both balls !



Collision "timescales"

- Collisions typically involve interactions that happen quickly compared to timescales used for our measurement before and after.

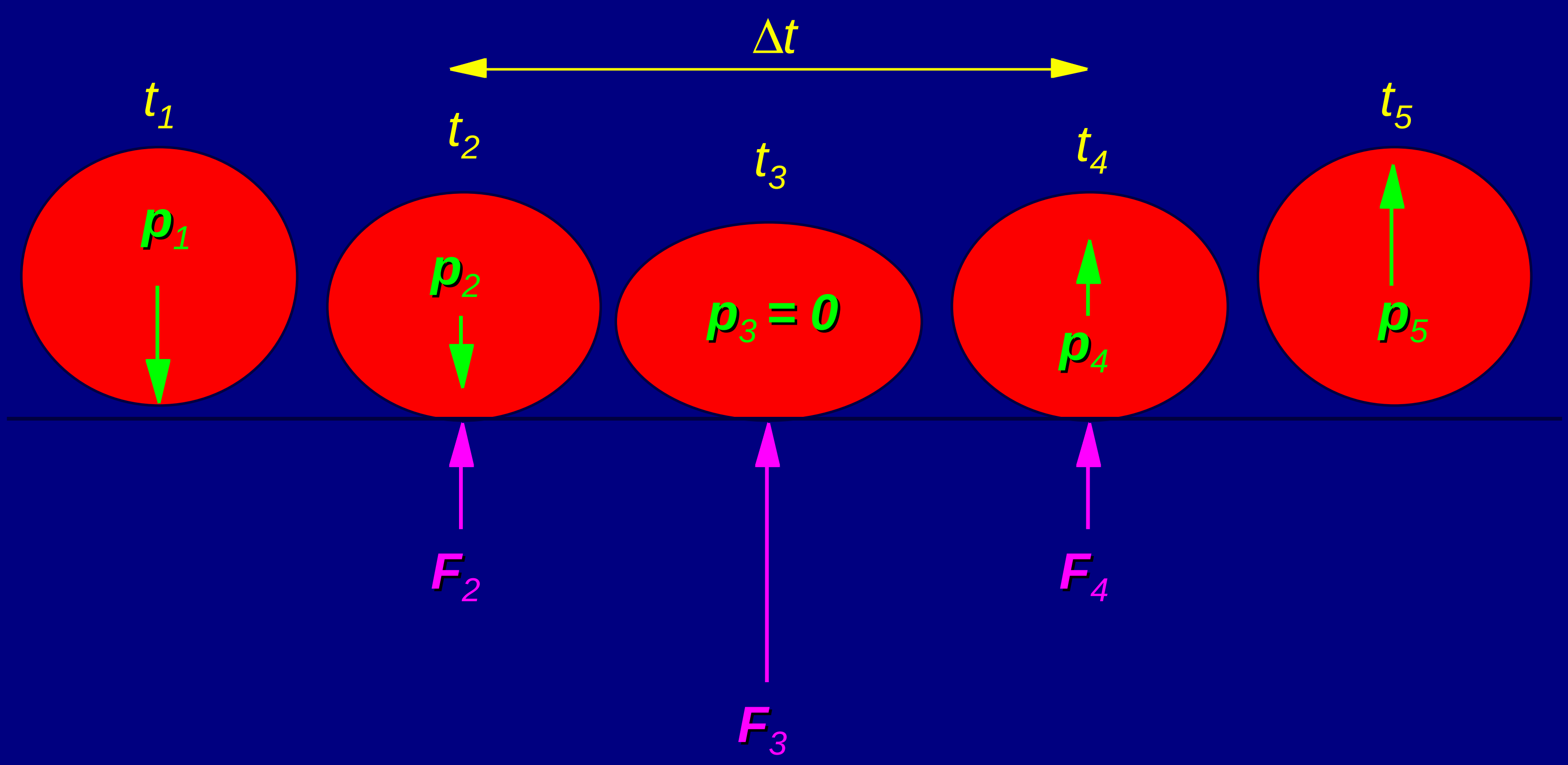


the balls are in contact

for a very short time.

Collision "timescales"

- During this brief time, forces involved can be quite large

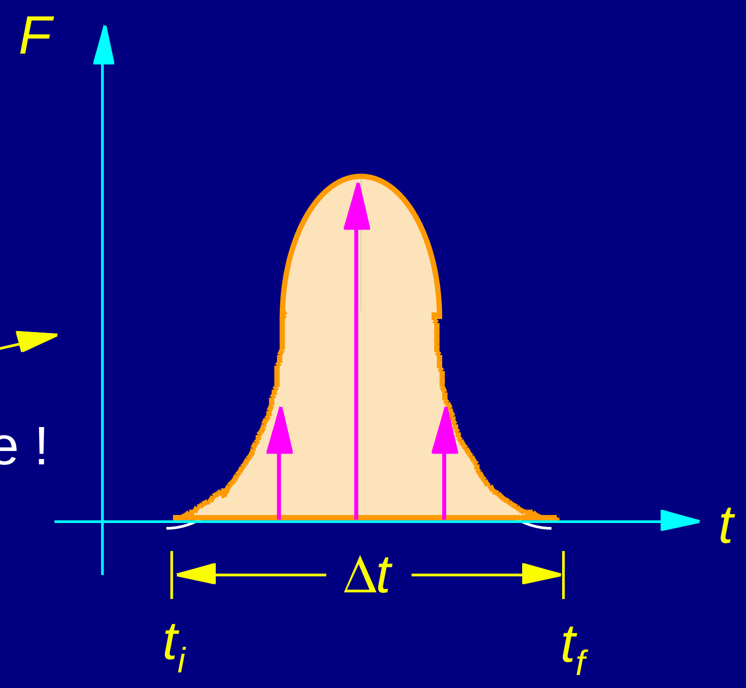


Force and Impulse

- The diagram shows the force vs time for a typical collision. The impulse, ***I***, of the force is a vector defined as the integral of the force during the collision.

$$\boldsymbol{I} = \int_{t_i}^{t_f} \boldsymbol{F}dt$$

Impulse ***I*** = area under this curve !



Force and Impulse

- If this "collision" force is much bigger than any other forces acting during this brief time, then to a good approximation:

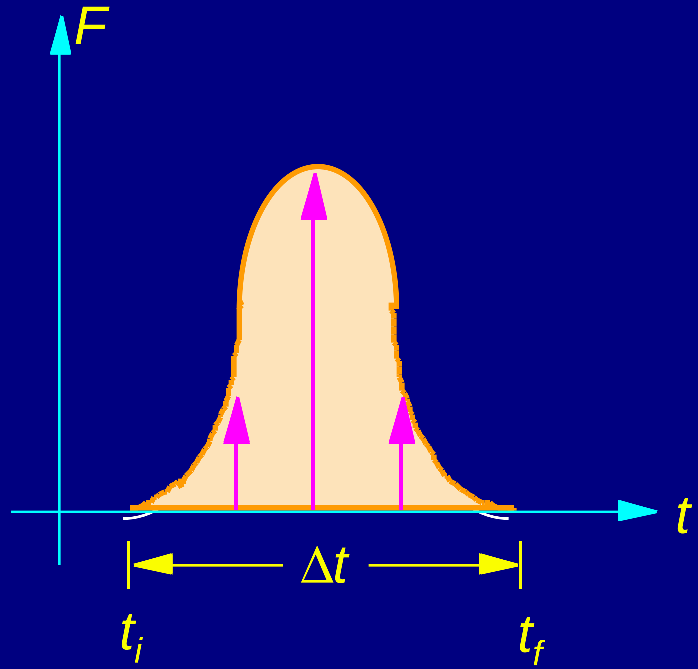
$$\boldsymbol{F} \cong \boldsymbol{F}_{NET} = \frac{d\boldsymbol{P}}{dt}$$

so the impulse becomes:

$$\boldsymbol{I} = \int_{t_i}^{t_f} \boldsymbol{F}dt = \int_{t_i}^{t_f} \frac{d\boldsymbol{P}}{dt}dt$$

$$= \int_{t_i}^{t_f} d\boldsymbol{P} = \boldsymbol{P}_f - \boldsymbol{P}_i \equiv \Delta\boldsymbol{P}$$

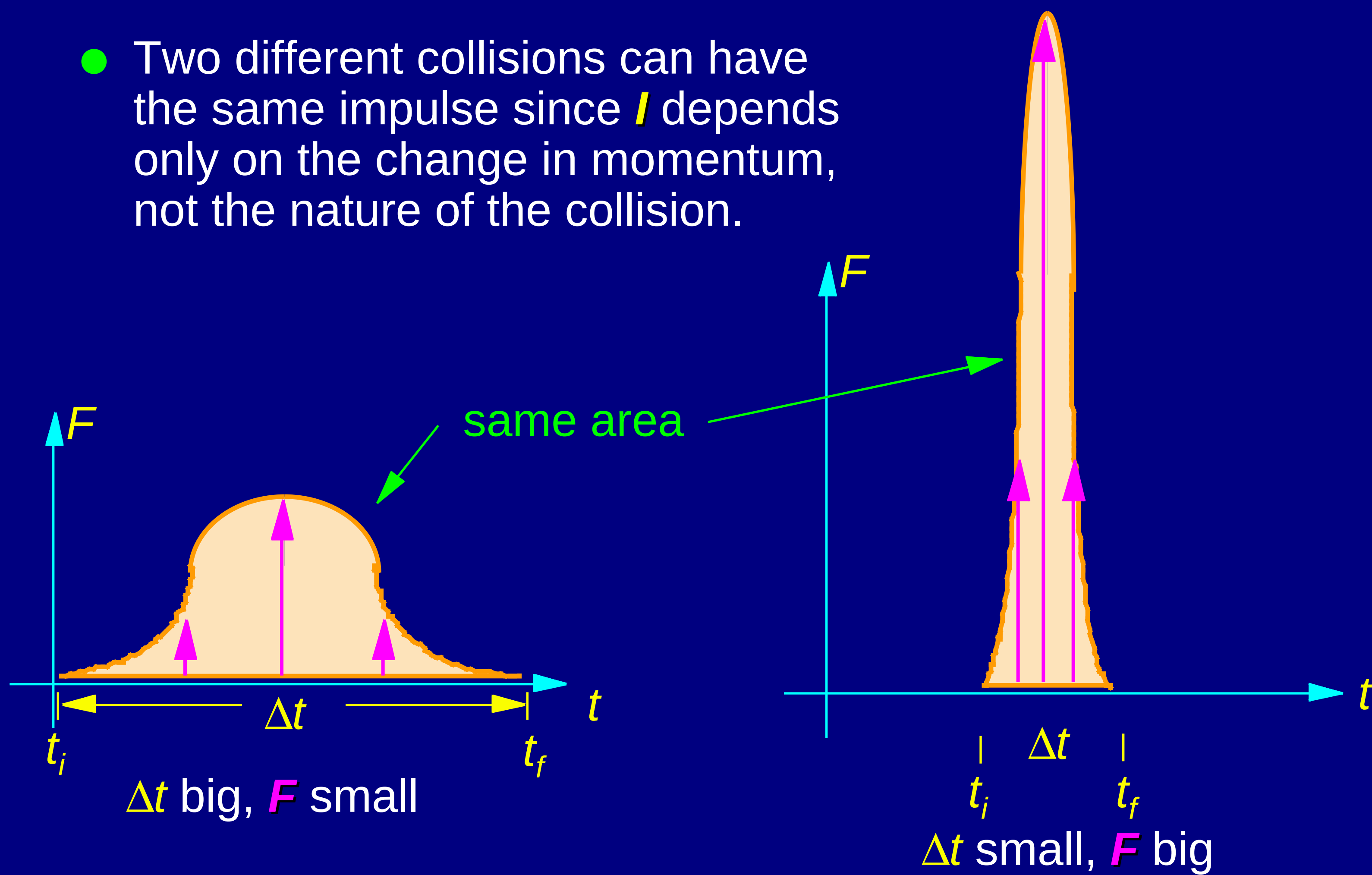
$$\boxed{\boldsymbol{I} = \Delta\boldsymbol{P}}$$



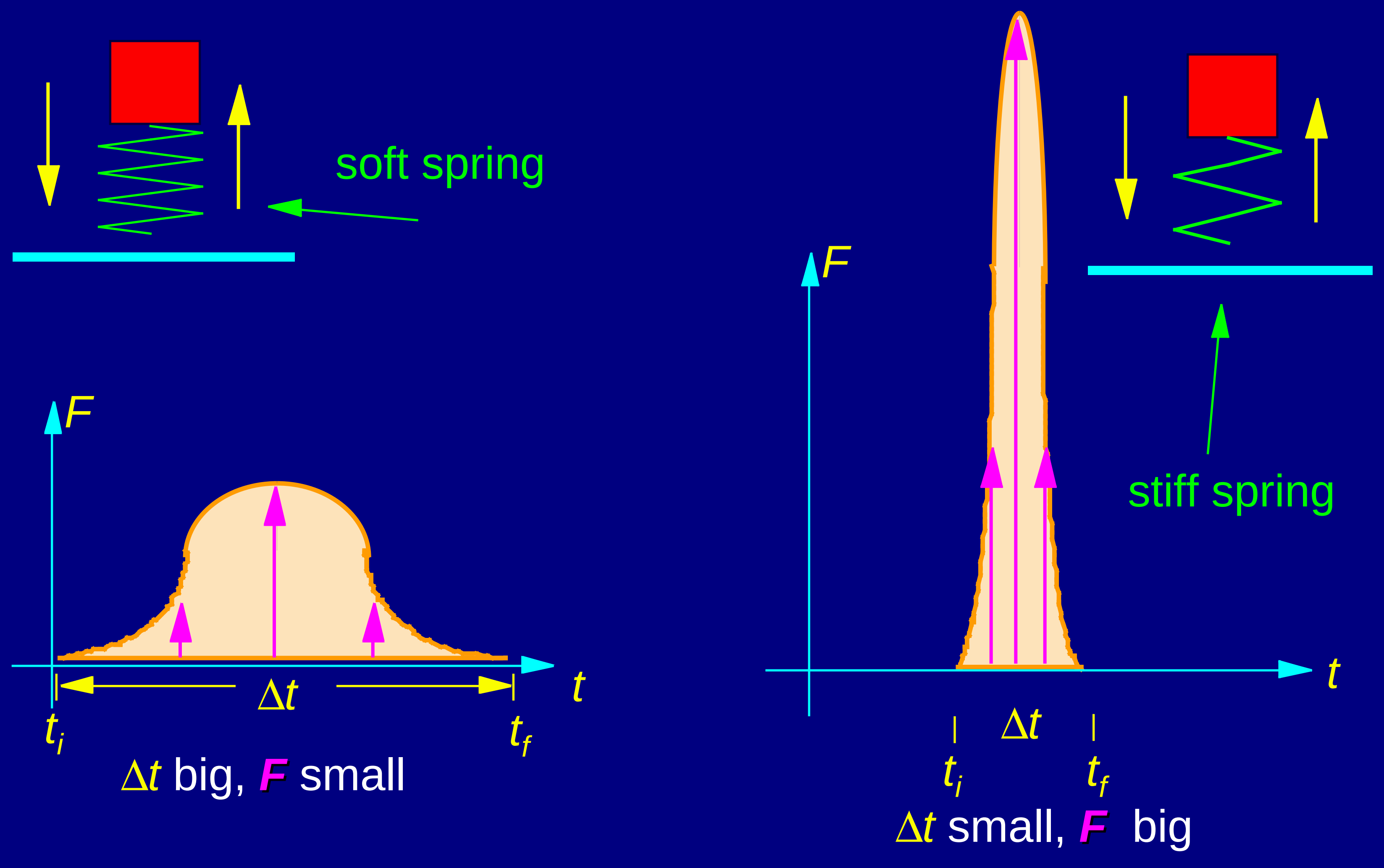
impulse = change in momentum !

Force and Impulse

- Two different collisions can have the same impulse since ***I*** depends only on the change in momentum, not the nature of the collision.



Force and Impulse



Force and Impulse

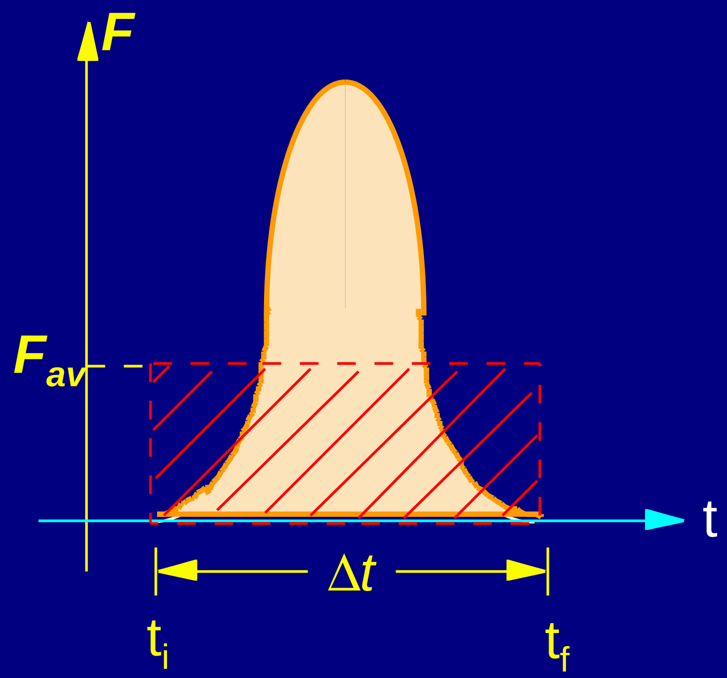
- We can use the notion of impulse to define "average force", which is a useful concept.

The time average of a force for the time interval $\Delta t = t_f - t_i$ is:

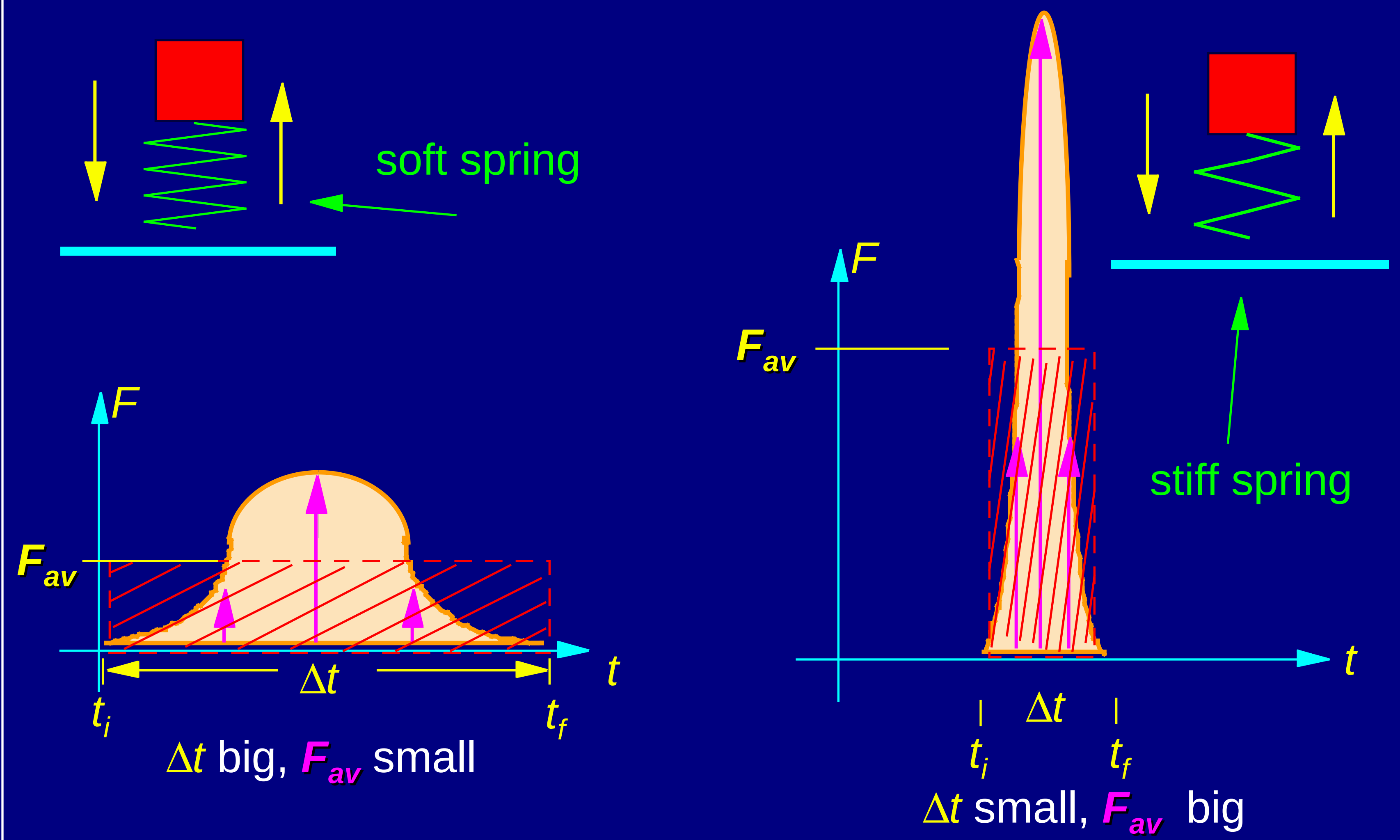
$$\boldsymbol{F}_{av} = \frac{1}{\Delta t}\int_{t_i}^{t_f} \boldsymbol{F}dt = \frac{\boldsymbol{I}}{\Delta t}$$

or:

$$\boldsymbol{F}_{av} = \frac{\Delta \boldsymbol{p}}{\Delta t}$$



Force and Impulse



VHS-Physics-SDW

Force and Impulse: Baseball Example

- A pitcher pitches the ball (*m = .7 kg*) at *145 km/hr* (about 90 mph).
- The batter makes contact with the ball for *.01 s* causing the ball to leave the bat going *240 km/hr* (about 150 mph).
- Find the net average force on the ball, disregarding gravity.

Baseball Example

First convert everything to m/s:

145 km/hr = 40.28 m/s

240 km/hr = 66.67 m/s

Next find the change in momentum (= the impulse):

$$P_f - P_i = (.7\ kg)(66.67\ m/s) - (.7\ kg)(-40.28\ m/s)$$

$$P_f - P_i = 74.87\ kg\text{-}m/s$$

Finally find the net average force:

$$\boxed{F_{av} = \frac{\Delta p}{\Delta t} = \frac{74.87\,Ns}{.01s} = 7487\,N}$$

Recap

- Two-dimensional elastic collisions.
- Examples (nuclear scattering, billiards).
- Impulse and average force.
- Work problems