Physics

Todays Agenda

- Inelastic collisions in one dimension.
- Inelastic collision in two dimensions.
- Explosions.
- Ballistic pendulum.
- Comment on energy conservation.

Momentum Conservation Review

$$\boldsymbol{F}_{EXT} = \frac{d\boldsymbol{P}}{dt}$$

$$\boxed{\frac{d\boldsymbol{P}}{dt} = 0}$$

$$\boldsymbol{F}_{EXT} = 0$$

- The concept of momentum conservation is one of the most fundamental principles in physics.
- This is a component (vector) equation.
  - We can appliy it to any direction in which there is no external force applied.
- You will see that we often have momentum conservation even when energy is not conserved.
- You will use it again and again to solve problems !!

Inelastic collision in 1-D: Example 1

- A block of mass *M* is initially at rest on a frictionless horizontal surface. A bullet of mass *m* is fired at the block with a muzzle velocity (speed) *v*. The bullet lodges in the block, and the block ends up with a speed *V*. In terms of *m,M*, and *V* :
  - What is the initial speed of the bullet *v* ?
  - What is the initial energy of the system ?
  - What is the final energy of the system ?
  - Is energy conserved ?

before after

See example 10-6

Example 1...

- Consiser the bullet & block as a system. After the bullet is shot, there are no external forces acting on the system in the x-direction **Momentum is conserved in the x direction !**

  - $P_{x,before} = P_{x,after}$
  - $mv = (M+m)V$

$$\Rightarrow \boxed{v = \left(\frac{M+m}{m}\right)V}$$

before

after

Example 1...

- Now consider the energy of the system before and after: (ignore gravity...its effect will be very small).
- Before:

$$E_B = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{M+m}{m}\right)^2 V^2 = \frac{1}{2}\left(\frac{M+m}{m}\right)(M+m)V^2$$

- After:

$$E_A = \frac{1}{2}(M+m)V^2$$

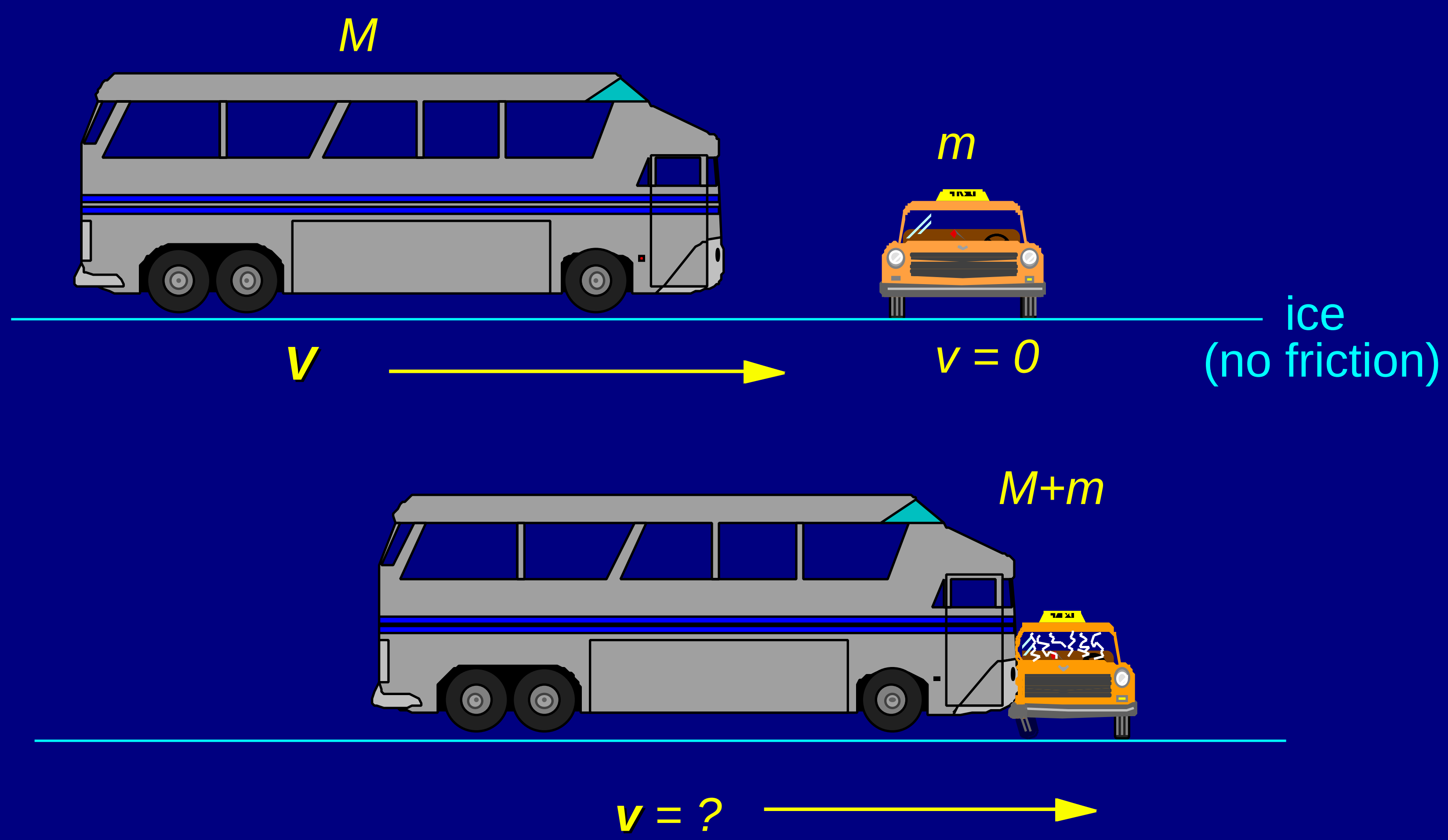
- So

$$\boxed{E_A = \left(\frac{m}{M+m}\right)E_B}$$

Energy is NOT conserved (friction stopped the bullet)

However momentum was conserved, and this was useful.

Inelastic Collision in 1-D: Example 2



Example 2...

Use conservation of momentum to find ***v*** after the collision.

Before the collision:

$$\boldsymbol{P}_i = M\boldsymbol{V} + m(0)$$

After the collision:

$$\boldsymbol{P}_f = (M+m)\boldsymbol{v}$$

Conservation of momentum:

$$\boldsymbol{P}_i = \boldsymbol{P}_f$$

$$M\boldsymbol{V} = (M+m)\boldsymbol{v}$$

$$\boxed{\boldsymbol{v} = \frac{M\boldsymbol{V}}{(M+m)}}$$

vector equation

Example 2...

- Now consider the energy of the system before and after:
- Before:

$$E_{BUS} = \frac{1}{2}MV^2 = \frac{1}{2}M\left(\frac{M+m}{M}\right)^2 v^2 = \frac{1}{2}\left(\frac{M+m}{M}\right)(M+m)v^2$$

- After:

$$E_A = \frac{1}{2}(M+m)v^2$$

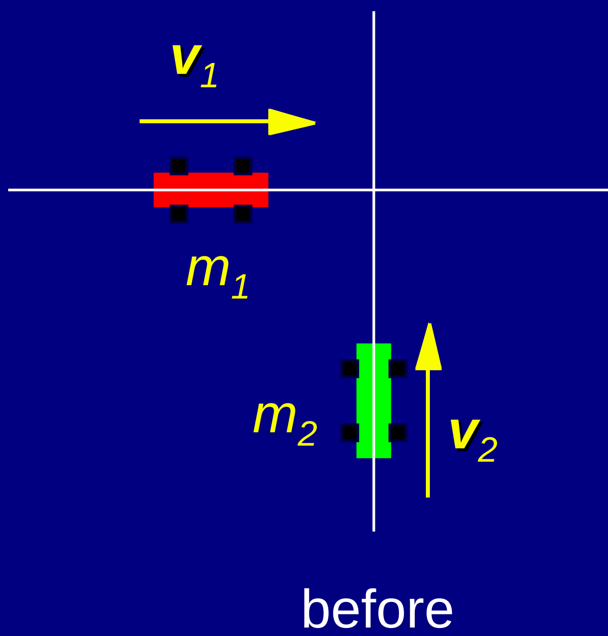
- So

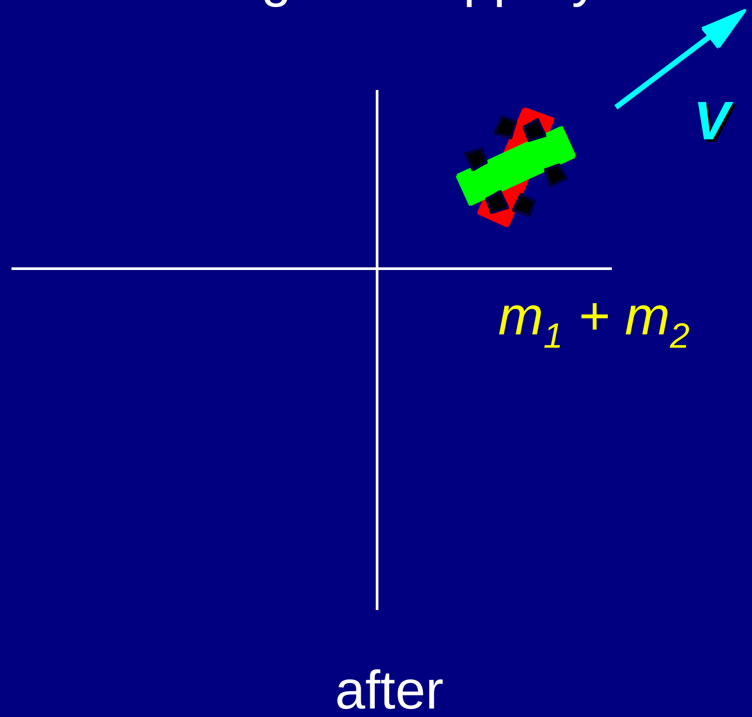
$$\boxed{E_A = \left(\frac{M}{M+m}\right)E_B}$$

Energy is NOT conserved in an inelastic collision !

Inelastic collision in 2-D

- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).



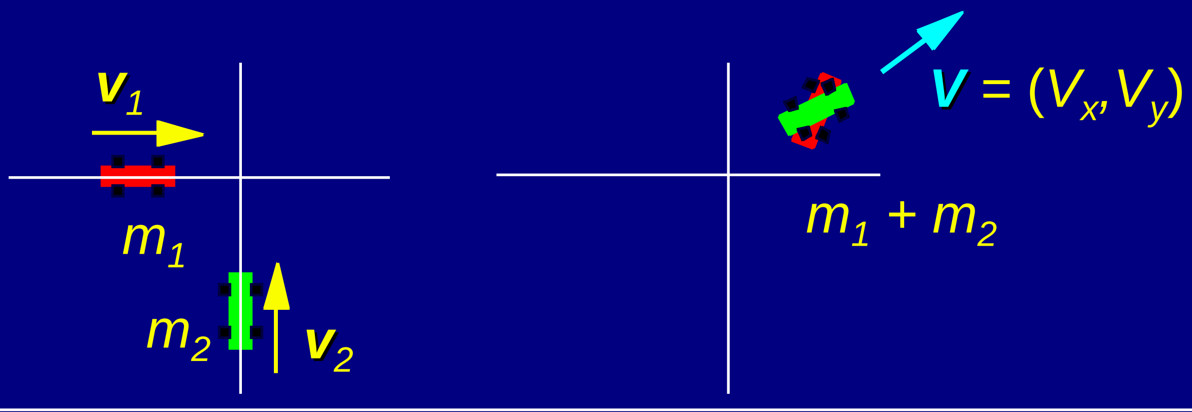


Inelastic collision in 2-D...

- There are no net external forces acting.
  - Use momentum conservation for both components.

$$P_{x,i} = P_{x,f} \Rightarrow m_1 v_1 = (m_1 + m_2) V_x \Rightarrow V_x = \frac{m_1}{(m_1 + m_2)} v_1$$

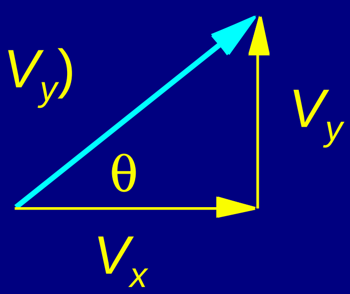
$$P_{y,i} = P_{y,f} \Rightarrow m_2 v_2 = (m_1 + m_2) V_y \Rightarrow V_y = \frac{m_2}{(m_1 + m_2)} v_2$$

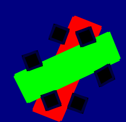


Inelastic collision in 2-D...

- So we know all about the motion after the collision !

$\boldsymbol{V} = (V_x, V_y)$





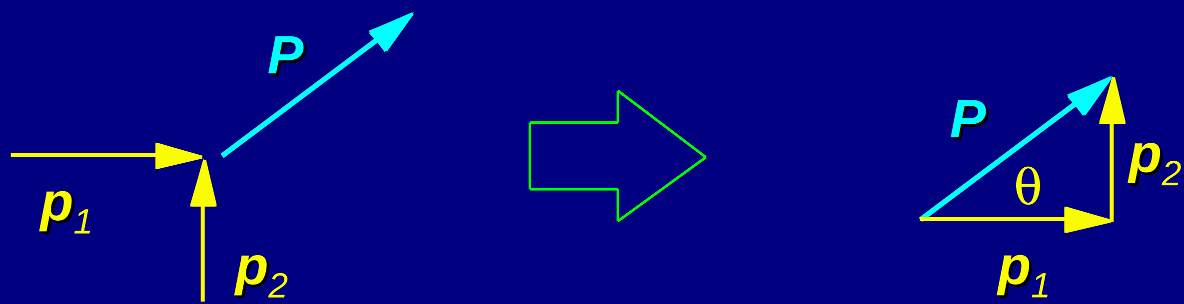
$$V_x = \frac{m_1}{(m_1 + m_2)} v_1$$

$$V_y = \frac{m_2}{(m_1 + m_2)} v_2$$

$$\tan\theta = \frac{m_2 v_2}{m_1 v_1} = \frac{p_2}{p_1}$$

Inelastic collision in 2-D...

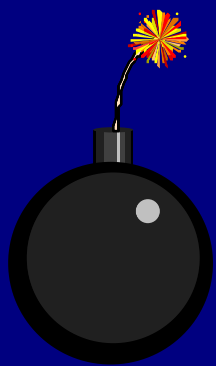
- We can see the same thing using vectors:



$$\tan\theta = \frac{p_2}{p_1}$$

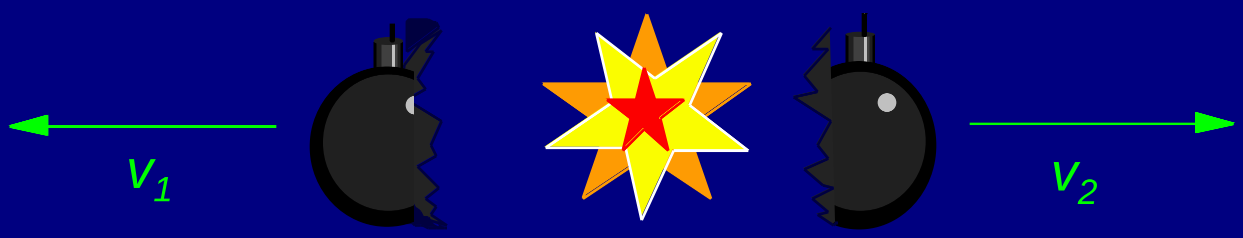
Explosion (inelastic un-collision)

Before the explosion:



M

After the explosion:

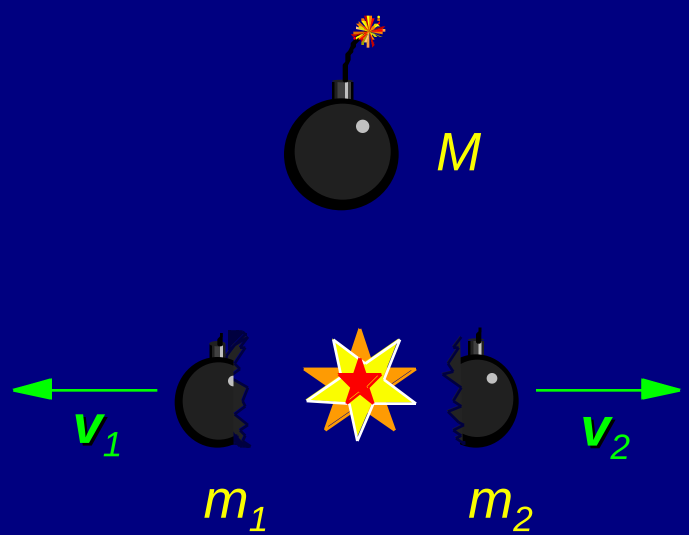


m_1 m_2

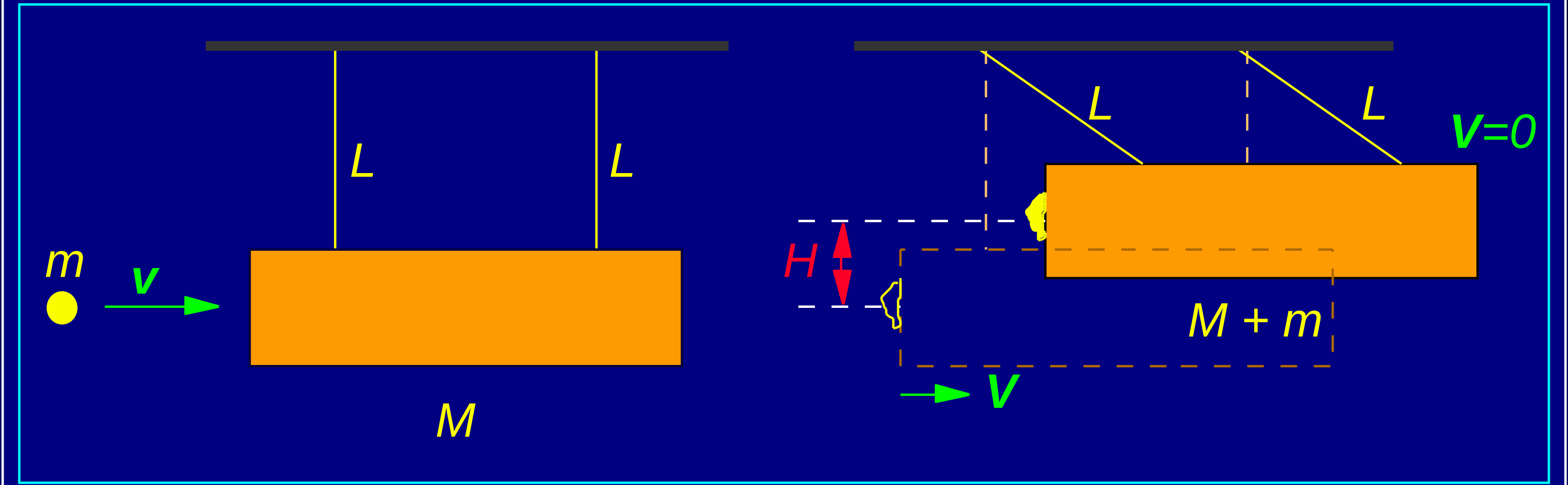
Explosion...

- No external forces, so ***P*** is conserved.
- Initially: $\mathbf{P} = 0$
- Finally: $\mathbf{P} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$

$$\boxed{m_1\mathbf{v}_1 = - m_2\mathbf{v}_2}$$



Ballistic Pendulum



- A projectile of mass *m* moving horizontally with speed *v* strikes a stationary mass *M* suspended by strings of length *L*. Subsequently, *m* + *M* rise to a height of *H*.

Given *H*, what is the initial speed *v* of the projectile?

Ballistic Pendulum...

- Two stage process:

1. *m* collides with *M*, inelastically. Both *M* and *m* then move together with a velocity ***V*** (before having risen significantly).

2. *M* and *m* rise a height *H*, conserving energy *E*. (no non-conservative forces acting after collision)

Ballistic Pendulum...

- Stage 1: Momentum is conserved

in x-direction: $mv = (m+M)V$

$$\boxed{V = \left(\frac{m}{m+M}\right)v}$$

- Stage 2: Energy is conserved

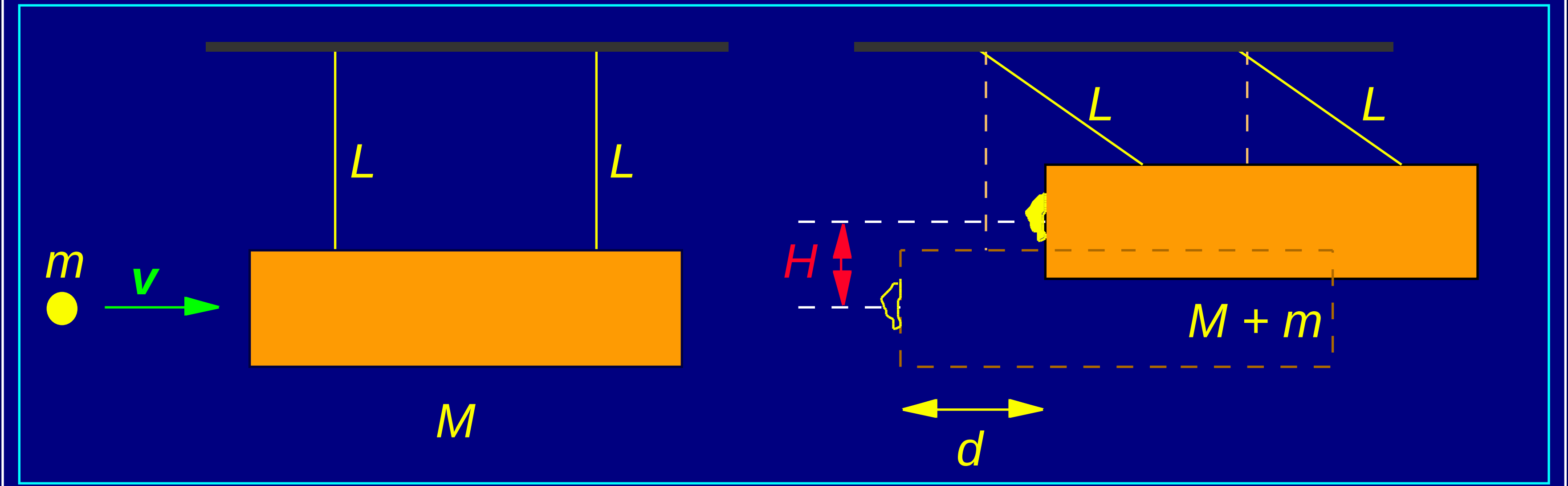
$$(E_I = E_F)$$

$$\tfrac{1}{2}(m+M)V^2 = (m+M)gH$$

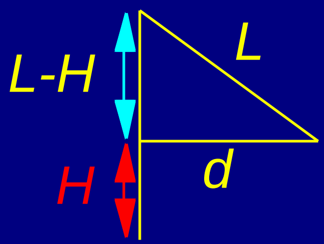
$$\boxed{V^2 = 2gH}$$

Eliminating **V** gives: $$\boxed{v = \left(1+\frac{M}{m}\right)\sqrt{2gH}}$$

Ballistic Pendulum Demo



- In the demo we measure foreward displacement *d*, not *H*:

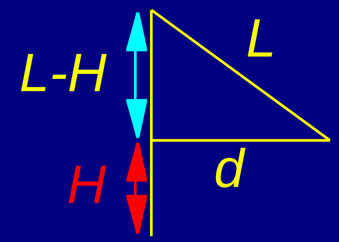


$$L^2 = d^2 + (L - H)^2$$

$$H = L - \sqrt{L^2 - d^2}$$

Ballistic Pendulum Demo...

$$H = L - \sqrt{L^2 - d^2} \approx \frac{d^2}{2L} \quad \text{if} \quad \frac{d}{L} << 1$$



$$v = (1 + \frac{M}{m})\sqrt{2gH} \Rightarrow \boxed{v = (1 + \frac{M}{m}) \cdot d \cdot \sqrt{\frac{g}{L}}}$$

for $d << L$

Lets see who can throw fast...

Comment on Energy Conservation

- We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
  - Energy is lost:
    - Heat (bomb)
    - Bending of metal (crashing cars)
- Energy **is not** conserved since **work** is done during the collision !
- Momentum along a certain direction **is** conserved when there are no external forces acting in this direction.
  - In general, easier to satisfy than energy conservation.

Recap

- Inelastic collisions in one dimension.
- Inelastic collision in two dimensions.
- Explosions.
- Ballistic pendulum.
- Comment on energy conservation.
- Work the problems